# Stigmergy as a Universal Coordination Mechanism: components, varieties and applications

Francis Heylighen Evolution, Complexity and Cognition group Vrije Universiteit Brussel

Abstract: The concept of stigmergy has been used to analyze self-organizing activities in an ever-widening range of domains, including social insects, robotics, web communities, and human society. Yet, it is still poorly understood, and as such its full power remains underappreciated. The present paper clarifies the issue by defining stigmergy as a mechanism of indirect coordination in which the trace left by an action in a medium stimulates a subsequent action. It then analyses the fundamental concepts used in the definition: action, agent, medium, trace and coordination. Stigmergy enables complex, coordinated activity without any need for planning, control, communication, simultaneous presence, or even mutual awareness. This makes the concept applicable to a very broad variety of cases, from chemical reactions to individual cognition and Internet-supported collaboration in Wikipedia. The paper classifies different varieties of stigmergy according to general aspects (number of agents, scope, persistence, sematectonic vs. marker-based, and quantitative vs. qualitative), while emphasizing the fundamental continuity between these cases. This continuity can be understood from a non-linear dynamics that lets more complex forms of coordination evolve out of simpler ones. The paper concludes with two specifically human applications, cognition and cooperation, suggesting that without stigmergy these phenomena may never have evolved in the first place.

# Past, present and future of the "stigmergy" concept

The concept of *stigmergy* was introduced by the French entomologist Pierre-Paul Grassé (Grassé, 1959) to describe a mechanism of coordination used by insects. The principle is that work performed by an agent leaves a trace in the environment that stimulates the performance of subsequent work—by the same or other agents. This mediation via the environment ensures that tasks are executed in the right order, without any need for planning, control, or direct interaction between the agents. The notion of stigmergy allowed Grassé to solve the "coordination paradox" (Theraulaz & Bonabeau, 1999), i.e. the question of how insects of very limited intelligence, without apparent communication, manage to collaboratively tackle complex projects, such as building a nest.

The insight came from Grassé's observation of how termites repair their nest. He noted that initially termites wander around more or less randomly, carrying mud and depositing it here or there. However, the deposits that are created in this haphazard way then stimulate the insects to add more mud in the same place. Thus, the small heaps quickly grow into columns that eventually come together to form an intricate cathedral of interlocking arches. The only communication between the termites is indirect: the partially executed work of the ones provides information to the others about where to make their own contribution.

Another classic example of stigmergy can be found in the pheromone trails left by ants that come back from a food source (Sumpter & Beekman, 2003). The pheromone stimulates other ants to follow the same path. When they find food, they too will reinforce the pheromone trail while following the trail back to the nest. This mechanism leads to the emergence of an efficient network of trails connecting the nest via the shortest routes to all the major food sources.

Up to about 1990, the notion of stigmergy appears to have remained limited to a small circle of researchers studying the behavior of social insects. However, one of these insect specialists, Jean-Louis Deneubourg, was also a member of the "Brussels School" of complex systems, headed by the late Nobel Prize in chemistry, Ilya Prigogine. In this interdisciplinary environment, it became clear that stigmergy was a prime example of spontaneous ordering or *self-organization* (Camazine et al., 2003; Deneubourg, 1977), and as such potentially applicable to complex systems other than insect societies.

With the advent of the agent-based paradigm in computer simulation, insect societies were conceptualized as *swarms* of simple agents that are able to perform complex tasks using various forms of self-organization, and especially stigmergy (Deneubourg, Theraulaz, & Beckers, 1992). The general ability to tackle complex problems exhibited by such self-organizing multi-agent collectives became known as *swarm intelligence* (Bonabeau, Dorigo, & Theraulaz, 1999; Kennedy, 2006). One class of stigmergic mechanisms in particular, so-called *ant algorithms*, turned out to be surprisingly powerful in tackling a variety of computational problems, including the notorious traveling salesman problem (Dorigo, Bonabeau, & Theraulaz, 2000) and the optimization of packet routing along communication networks (Kassabalidis et al., 2001).

A similar stigmergic mechanism was recently recognized in molecular biology (Tabony, 2006) to explain the self-organization of the microtubules that support many functions in the cell. These microscopic tubes change shape and move by absorbing tubulin proteins at one end, and releasing them at the other end. The "trail" of tubulin left at the shrinking end attracts the growing ends of other microtubules, resulting in the formation of a coherent "wave" of microtubules moving in the same direction.

Stigmergy was applied not only to software agents, but to their hardware analogues: autonomous robots. Groups of very primitive robots proved able to tackle non-trivial tasks, such as clustering items in different groupings, in a way similar to ants (Beckers, Holland, & Deneubourg, 1994; Deneubourg et al., 1991; O. Holland & Melhuish, 1999). These robotic implementations inspired the application of stigmergic

models to problems of coordination and control in manufacturing (Valckenaers, Van Brussel, Kollingbaum, & Bochmann, 2006). After this expansion of the stigmergy concept from social insects to the domains of artificial life, artificial intelligence and behavior-based robotics, a perhaps obvious next step was computer-supported collaboration between human agents, in particular via the world-wide web (Dron, Boyne, & Mitchell, 2001; Heylighen, 1999; Elliott, 2007; Bolici, Howison, & Crowston, 2009).

A prototypical example is Wikipedia, the free web encyclopedia which has grown to become the largest one in existence thanks to the fact that every reader is stimulated to improve and expand the writings of previous contributors (Heylighen, 2007). A similar dynamics of contributions building further on previous contributions characterizes opensource software development (Bolici et al., 2009; Robles, Merelo, & Gonzalez-Barahona, 2005). But it quickly became clear that human collaboration does not need computer support to profit from stigmergy (Parunak, 2006; Elliott, 2007). Probably the best-known example of stigmergic self-organization is the "invisible hand" of the market: the actions of buying and selling leave a trace by affecting the price of the transacted commodities. This price in turn stimulates further transactions. Via the related conceptions of distributed cognition and the extended mind, stigmergy has now also started to make its mark on theories of cognition and epistemology (Marsh & Onof, 2008; Ricci, Omicini, Viroli, Gardelli, & Oliva, 2006; Susi & Ziemke, 2001).

It is clear that since 1990, the concept of stigmergy has undergone a rapid diffusion across an ever-growing number of application domains. While the number of publications that mention the term "stigmergy" appears to have remained roughly constant at about 1 per year in between 1960 and 1990, the following years witnessed an impressive exponential growth in that number: from 3 in 1991 to about 500 in 2006 (as measured via a search on scholar.google.com). The growth then slowed down, reaching about 700 in 2013. However, to me it seems likely that this is still merely the first stage of an on-going development. The as yet underinvestigated application of stigmergy to human affairs opens the way to a virtually limitless expansion across the various scientific, technological and social disciplines that study society, cognition, and behavior.

I contend in this paper that the potential for theoretical explanation and practical application of the stigmergy concept is much larger still than hitherto assumed. What (Parunak, 2006) noted about human institutions, that the more difficult issue is to find examples where stigmergy does *not* apply, extends to complex systems in general, and in particular to systems that exhibit some form of cognition, cooperation, or organization that is the result of evolution. When properly defined, the mechanism of stigmergy appears to be nearly ubiquitous, and able to illuminate a variety of conceptual problems in a non-trivial manner.

The matter of definition, however, is crucial to a proper understanding and application. Definitions in the literature tend to be vague, restricted in scope, and mutually incoherent (Dipple, Raymond, & Docherty, n.d.; Shell & Mataric, 2003). Misunderstandings have arisen particularly because of a confusion between the general notion of stigmergy and its specific instantiation in ant algorithms, i.e. the reinforcement

with pheromones of frequently traveled paths by virtual "ants". The depositing of pheromone traces is an example of what we will call quantitative, marker-based stigmergy (Parunak, 2006). Stigmergy in the most general sense does not require either markers or quantities. Another, even more common misunderstanding is that stigmergy only concerns groups or swarms consisting of many agents. As we will show, stigmergy is just as important for understanding the behavior of a single individual.

The next section of this paper will clarify the meaning of stigmergy, propose an unambiguous definition, and summarize its benefits in explaining spontaneous forms of coordination. We will then go into greater depth concerning the different components and aspects of the mechanism. This will allow us to situate and classify the apparently very different forms of stigmergy, while remaining focused on their common core.

Perhaps a last question to conclude this introductory section: if stigmergy is so fundamental, ubiquitous and explanatorily powerful, then why has it taken so long for it to be recognized? The more obvious answer is that the study of termites that gave rise to this conception is a very specific discipline with no evident applications to other sciences. Moreover, the defining publication (Grassé, 1959), appearing in French in a specialized journal, obviously only reached a limited audience. Its reach appears to have widened significantly only after Deneubourg, a researcher active in both French and English, and the domains of both social insects and self-organizing systems, started applying the concept outside of its original context.

But why did not someone else come up with this simple and elegant notion? A more fundamental answer is that stigmergic interaction is by definition indirect, while our mind is biased to look for direct causes of the phenomena we observe. If we note that agents act in a coordinated way, our natural inclination is to seek the cause of one agent's behavior directly in another agent's behavior, assuming that there is an immediate communication from the one to other. Failing to find this link, we assume that the agents are driven by the same cause, such as a shared instinct, plan, or leader that controls their behavior. We do not spontaneously consider the option that one agent may drive another agent's behavior only via the indirect route of an unintentional trace left in a passive environment.

Finally, we tend to assume that intelligent organization must be produced by an intelligent agent, as illustrated by Paley's well-known watchmaker argument (Dawkins, 1996). Darwin's theory of *natural selection* provides a mechanism that can generate complex organization without presupposing intelligence. However, its counter-intuitive nature may be illustrated by the fact that, in spite of massive empirical evidence, it is still being challenged by the "intelligent design" school (Behe, 2009). Another proposed mechanism to generate coordinated activity, *self-organization*, is much more recent, and is still far from being generally accepted and, even less, generally understood. I wish to suggest here that stigmergy is another type of mechanism for generating complex organization, but which has some distinct features of its own that may resolve some outstanding problems with these previously proposed explanations.

# The meaning of stigmergy

# From etymology to definition

The term "stigmergy" was derived by Grassé from the Greek roots, *stigma*, which means "mark or puncture" (typically referring to the tattoo used to mark slaves), and *ergon* which can mean "work, action, or the product of work". Grassé motivated this derivation by interpreting *stigma* as a goad, prod or spur, i.e. a stinging movement ("piqure" in the original French text) that incites activity. *Ergon* is then the result of previous work responsible for this stimulus or incitement. Thus, (Grassé, 1959) defined stigmergy as "the stimulation of workers by the very performances they have achieved" (from the original English abstract).

However, in a more recent review paper, (Parunak, 2006) proposes a different reading of the Greek etymology that is at least as compelling: if we interpret *stigma* as "mark" or "sign" and *ergon* as "action", then stigmergy is "the notion that an agent's actions leave signs in the environment, signs that it and other agents sense and that determine their subsequent actions". Summarizing, in Grassé's interpretation the product of work (*ergon*) functions as a stimulus (*stigma*) for action; in Parunak's interpretation, action (*ergon*) leaves a mark (*stigma*). While this double interpretation may seem to add to the confusion, it actually provides an elegant illustration of the bidirectional nature of stigmergy. The process described by both Grassé and Parunak is a *feedback loop*, where an action produces a mark which in turn incites an action, which produces another mark, and so on (see Fig. 1). In other words, actions stimulate their own continued execution via the intermediary of the marks they make—where a mark is a perceivable effect, trace or product of an action.

This brings me to my own definition:

stigmergy is an indirect, mediated mechanism of coordination between actions, in which the trace of an action left on a medium stimulates the performance of a subsequent action.



Fig. 1: the stigmergic feedback loop

#### Basic components of stigmergy

Let us analyze the different terms in this definition, and from thereon the conceptual components necessary to build a stigmergic process.

Most primitive is the concept of *action*, which I interpret as a causal process that produces a change in the state of the world. Normally, we assume that an action is performed by an *agent*, which is typically seen as an autonomous, goal-directed system. However, the concept of agent does not appear to be necessary for a definition of stigmergy: as we will see, the mechanism applies perfectly well to the coordination of actions performed by a single, unspecified agent, in which case there is no need to identify different agents. Moreover, further extensions of the stigmergy concept can even do away with the notion of agent altogether, and consider the coordination of "agentless" actions that are merely events or physical processes—such as chemical reactions. This views fits in with the ontology of action (Heylighen, 2011; Turchin, 1993), which sees action as the primitive element from which all other concepts are derived. The concept of agent remains useful, though, in cases where we wish to distinguish different agents able to perform different actions.

As causal processes, actions have an antecedent or cause, and a consequent or effect. In simple agent-based models used in artificial intelligence the antecedent is usually called *condition*, and the consequent simply *action*. The condition specifies the state of the world in which the action occurs, while the action specifies the subsequent transformation of that state. The causal relation is represented as a "production rule" or production, which consists of the simple relation:

*condition*  $\rightarrow$  *action* 

It is to be read as:

IF condition holds, THEN perform action

For example, a thermostat will obey the production rule: IF the temperature is below the goal temperature, THEN switch on the heating (Heylighen & Joslyn, 2003). While this reading seems to imply an elementary cognitive process of sensation or perception, to ascertain whether the condition holds, the notation is equally applicable to agentless, physical processes. For example, consider the following chemical reaction:

 $NaOH + HCl \rightarrow NaCl + H_2O$ 

The first part represents the necessary condition for the reaction to occur: an NaOH molecule and an HCl molecule must be simultaneously present. The second part of the reaction represents the product or result of the reaction: the formation of an NaCl molecule together with an H<sub>2</sub>O molecule. More generally, we can write such reactions in the following form (where the "+" operators represents a conjunction of the conditions or resources necessary for the process to take place):

 $a + b + \ldots \rightarrow x + y + \ldots$ 

Chemical Organization Theory (Dittrich & Fenizio, 2007) is a formal framework that shows how collections of such simple reactions can become coordinated by acting on a shared medium, the "reaction vessel", where they produce an evolving trace expressed by the concentrations of the different "molecules" (a, b, ...). This coordinated pattern of activity defines an "organization": a self-sustaining, dynamic network of interacting "molecules". While the inspiration for this model comes from chemistry, it is equally applicable to other kinds of abstract actions in which the agent is ignored—such as political and economic interactions (Dittrich & Winter, 2005, 2008)

According to our definition, the action part of a rule produces a change in the state of the world. This means that it creates a new condition, which may activate another condition  $\rightarrow$  action rule, and thus a new action. For example, the thermostat, by switching on the heating, will eventually produce the new condition "temperature high enough", which in turn will trigger the new action "switch heating off" (Heylighen & Joslyn, 2003). The NaCL molecule may react with another molecule in the solution and thus produce yet another compound. This triggering of an action by a previous action via the intermediary of its result is precisely what Grassé defined as stigmergy. Yet, the way we arrived at this notion is so simple and general that it merely requires a minimal assumption of causality. In the next sections, we will need to explain how such a simple mechanism can produce such rich and unexpected phenomena.

First, we should note that the causal relation does not need to be fully deterministic: in general, the condition is neither necessary nor sufficient for the action to occur. According to Grassé's definition of stigmergy, the condition merely *stimulates* the performance of the action. This means that the presence of the condition makes the performance of the action more probable. Formally:

P(action | condition) > P(action),

where P(A) is the general probability of A occurring, and P(A|B) the conditional probability of A occurring given that B is the case.

Note that in some cases, a condition may on the contrary *inhibit* an action, i.e. make it less likely. For example, the presence of a red light makes it less likely that someone would cross the street. In that case, we may still keep to the definition of "stimulation" above, simply by considering the opposite or negation of that condition as the stimulus: e.g. the disappearance of a red light makes it more likely that someone would cross the street.

We must now introduce another core component of stigmergic activity, the *medium*. The medium is that part of the world that undergoes changes through the actions, *and* whose states are sensed as conditions for further actions. The medium is a non-trivial entity, since many aspects of the world are either not affected by actions, or not perceivable as conditions for new actions. For example, while I can clearly see the clouds in the sky, no matter how hard I try, I cannot change their position. Vice-versa, I have the power to throw a rock in the sea, but I cannot see where that rock will end up. In either case, there is no basis for a stigmergic chain of actions triggering further actions. On the other hand, I can both perceive and affect the arrangement of sand on a beach, and this allows me to build an intricate sand castle via a coordinated sequence of condition  $\rightarrow$  action pairs. Neither the sea nor the sky is a stigmergic medium, but the beach is.

Note that most authors (e.g. Parunak, 2006) use the term "environment" for what I call "medium". This term is much less accurate, though. First, as noted, the environment is not in general both perceivable and controllable. Second, the environment normally denotes everything *outside* the system or agent under consideration. However, stigmergy can also make use of an *internal* medium. For example, different physiological processes in the body communicate via the release of hormones in the bloodstream (medium). This communication is indirect: e.g. the liver does not directly send a message to the brain; both merely "read" the hormonal messages deposited in the blood that irrigates both. More generally, many aspects of the agent's own state, such as the agent's position, speed and orientation, belong to the medium, since they are controllable and perceivable by self and others. In the example of the chemical reactions, the medium is the reaction vessel containing the solution in which the molecules are present.

Finally, if we conceive the environment as that part of the world that interacts with an agent, then different agents live in different environments or "Umwelts": not all phenomena perceivable or controllable by one agent are similarly perceivable and controllable by another agent. When we consider stigmergic coordination between different agents, we need to define the medium as that part of the world that is controllable and perceivable by all of them. This is necessary to ensure that the different agents can interact via the medium. The role of the medium is to allow interaction or communication between different actions, and thus, indirectly, between the agents that

perform the actions. It is this *mediating* function that underlies the true power of stigmergy (Heylighen, 2006).

A final component of a stigmergic system is the mark or *trace*, i.e. the perceivable change made in the medium by an action, which may trigger a subsequent action. I prefer the term "trace" because it can denote an unplanned or even undesired side effect of the action, unlike a "mark" which is normally made intentionally. As we will see, some forms of stigmergy rely on intentionally made signs or signals ("markers"), but in the most general situation, this is not the case. The trace is a consequence of the action, and as such, it carries information about the action that produced it. We might see the trace as a message deposited in the medium through which the pattern of activity communicates with itself, or maintains a continuously updated "memory" of its achievements. From the point of view of an individual agent, on the other hand, the trace is a *challenge*: a situation that incites action, in order to remedy a perceived problem or shortcoming, or to exploit an opportunity for advancement (Heylighen, 2012a).

# Goal-directed action

Let us assume that actions are performed by agents with a minimal form of intentionality, i.e. agents whose actions are appropriate to the conditions that trigger them, in the sense that they help the agent to move toward its (implicit or explicit) goals. This is an application of what Dennett (1989) called the "intentional stance".

This assumption is less strong than it may seem, since natural agents (such as living organisms) have the implicit goal of fitness (i.e. survival and reproduction) built into them by natural selection, while artificial agents (such as thermostats or robots) have their goals specified by their designers. Even natural, non-living objects, such as stones or molecules, can be seen as goal-directed, in the sense that their dynamics can always be modeled as trying to optimize some function of their state (Heylighen, 2011; Mesarović & Takahara, 1975) (e.g. potential energy).

This assumption allows us to add a "virtual" component to the stigmergic mechanism, i.e. a component that in a sense only exists for the observer: the *tasks* that the stigmergic system is to perform. Since a stigmergic system does not plan, it generally does not have any awareness or representation of the tasks, jobs or duties that it still has to carry out. But for the outside observer, it may be helpful to use the term "tasks" as shorthand for the actions that are to be performed. A *task* can be defined as an action that:

- 1) is required to achieve the agents' goals;
- 2) is not yet performed;
- 3) can be performed on the present medium once the right conditions arise.

This minimal intentionality means that actions are not random or blind, like the mutations that underlie biological evolution, but generally produce some improvement in the agent's situation, i.e. movement closer to the goal. Reaching a far-away goal, however, requires more than a minimal intelligence: this will typically necessitate a complex, coordinated scheme of actions, performed according to a specific order or logic.

The difficulties involved in problem solving, planning, and project management may remind us that there is no simple or obvious way to go from elementary actions to complex activity schemes. This brings us to the problem of coordination (Crowston, 1997), which stigmergy appears to solve.

# Coordination

According to the Oxford Dictionary, *coordination* can be defined as:

the organization of the different elements of a complex body or activity so as to enable them to work together effectively.

In the case of stigmergy, the "elements" are the different actions or agents. "Effectively" means that they achieve an intended effect or goal. "Working together" means that the actions are harmonious or synergetic, the one helping rather than hindering the other. "Organization" can be defined as structure with function (Gershenson & Heylighen, 2005). The function is the achievement of the intended effect. A "structure" consists of distinct elements (the actions or agents) that are connected in such a way as to form a coherent whole. This brings us to focus on the *connections* that integrate the actions into a synergetic, goal-directed whole.

According to coordination theory (Crowston, 1997), we can distinguish the following fundamental dependencies or connections between actions or processes:

- 1) one action can be *prerequisite* for the next action: the product or output of the first is a necessary condition or input for the second. This determines the *sequential* organization of the process, or *workflow*, where activity moves step-by-step through a sequence of tasks (what needs to be done next?).
- 2) two actions can require the same condition (input) and/or contribute to the same effect or goal (output), i.e. they are performed in *parallel*. This determines the *allocation of resources* (who receives what?) and the *division of labor* between agents (who is to do what?).

Effective coordination means that the right actions are performed by the right agents at the right time and place. Let us consider the building of a house as an activity that requires coordination between its different tasks. The task of laying electricity obviously can only be performed once the windows and roof are installed. Roofing is therefore prerequisite for laying electricity, and the electricians will have to wait until the roofers are finished. Plastering the interior walls, on the other hand, can only be done after the electrical cables and outlets have been dug into the walls. This implies the sequence: roofing  $\rightarrow$  laying electricity  $\rightarrow$  plastering. On the other hand, plumbing and laying electricity can be performed simultaneously or in parallel, since they both require roofing and are prerequisite for plastering, but are otherwise independent of each other. The

dependencies or connections between these different processes can be represented in the following "workflow" diagram (Fig. 2).



Fig. 2: parallel and sequential connections between tasks

This diagram represents only a small part of the complex of activities that is necessary to construct a building. Construction work and other complex activities are normally planned in detail beforehand, using tools such as project schedules and GANTT charts, to specify the dependencies between the different tasks. This planning is necessary to make sure that the work is efficiently performed, by avoiding situations such as the plasterers turning up when the plumbing is still going on so that they cannot start their work. The plan will normally specify the beginning and end of all the actions as well as the agents that are to perform them, and possibly the places or resources that the agents need to access. If everybody keeps to this plan, the plasterers will show up on the exact time and place that the plumbers are supposed to have finished their work.

The problem with planning, of course, is that there will always be unforeseen contingencies, such as the plumbers needing an extra day to finish their work, or, on the contrary, finishing two days early. In both cases, the work is performed less efficiently than it could be, either because the plasterers need to go home because the plumbing is not ready yet, or stay home waiting for the work to finish when they could already have started. Contingencies disturbing carefully laid-out plans can have even worse results, as illustrated by the following joke:

A pensioner watches two city workers busy in the municipal park. The one digs a series of deep holes at regular intervals. The other one then shovels the mounds of earth carefully back into each hole, and flattens the soil. The pensioner asks him: "Isn't that a waste of effort what you are doing?", to which the worker replies: "No, we always work this way, and it is very efficient. It is just that the third guy who plants the trees did not show up today."

One way to deal with such contingencies is to let the agents communicate about their work. For example, the plumbers finishing early or late could call the plasterers to warn

them about the different finishing date. However, this assumes that agents know all other agents that depend on their work, and have a general notion of what these dependencies are so that they can improvise or reschedule their activity in the light of the new information. With complex activities, this is tricky and can easily lead to misunderstandings or confusions that make things worse. An alternative is that all agents report to a supervisor, who keeps track of the plan, reschedules if need be, and warns everyone involved of the changes. However, such central controller becomes a bottleneck that is even more sensitive to disturbances, creating the risk that the whole plan falls apart because the supervisor is not available to pass on reschedules.

# The benefits of stigmergy

How does stigmergy solve the problem of coordination? In the examples above, the different agents would regularly check the situation at the work site, and as soon as they encounter the right conditions, they would start their work. For example, once the plumbers observe that the roof and windows are in place, they would start plumbing. Simultaneously but independently, the electricians would do their job. The plasterers would begin as soon as *both* the plumbing and the electricity are finished. On the other hand, the municipal worker would fill a hole only on the condition that it contains a tree.

While this approach may seem natural for termites, who are anyway all wandering around their nest building site, you might wonder whether it would not be inefficient to demand that specialized workers visit the site every day while they are not yet needed. However, this can be easily tackled with modern technology, by providing a website on which the state of the work is registered in real time. In this way, the plumbers can see immediately whether they are needed, without losing time traveling to the site. The website plays the role of a medium providing special markers to guide the execution of the work—similar to the pheromones used by ants.

This is in essence how a community of programmers residing in different parts of the world collaboratively develop a complex suite of open-source software: they regularly check their shared website for new modules, updates, requests for features, or postings of bugs. They address these challenges by writing additional code or suggesting solutions, posting these results in turn on the website for others to see and to elaborate (Bolici et al., 2009; Heylighen, 2007; Heylighen, Kostov, & Kiemen, 2013). Such open source development has proven to be at least as effective as the traditional planned software development performed in large corporations (Weber, 2004), without requiring any central supervision or other complicated arrangements (Raymond, 1999).

Perhaps the only disadvantage compared to a perfectly designed and executed plan, is that the stigmergic approach does not guarantee an optimal use of the "workforce". While the roofers are working, the plumbers must either wait or perform another task. If they are busy with another task, there is no guarantee that it will be finished exactly when the roofers finish their job. This suboptimal use of workers can be minimized by creating a pool of available workers (much) larger than needed for this

particular job, so that together they can keep track of several jobs in parallel. Assuming that the tasks do not all start at the same time, there would always be some workers available for any job that opens up, without requiring workers to wait long times in between jobs, or tasks to wait long for workers to execute them. This is the approach underlying the job ticketing systems used in call centers (Heylighen & Vidal, 2008; Orrick, Bauer, & McDuffie, 2000), but also the one used by ants and termites. It explains why the best-known applications of stigmergy typically rely on "massive parallelism", i.e. many agents active simultaneously (Manderick & Moyson, 1988). In the case of software development, there is no particular time at which a coding job has to be done, so programmers can be flexible in deciding when to produce an improvement suggested by the stigmergic repository of tasks and products.

With such a stigmergic organization, no conflicts between instructions and reality arise, no needless delays occur and no effort is wasted—whatever the contingencies that may disturb the plan. Moreover, this solution is perfectly robust, and independent of any errors in communication or control. It also does not depend on the number of agents, tasks, or dependencies between tasks. This allows it to scale up to tasks of indefinite sizes. The only requirements are that the agents can recognize the right conditions to start their work, and that they can all access the medium in which these conditions are registered. In summary, stigmergy provides an extremely simple and reliable solution to a problem that is potentially unlimited in complexity.

Compared to traditional methods of organization, stigmergy makes absolutely minimal demands on the agents. In particular, in stigmergic collaboration there is *no need for*:

- **planning or anticipation**: agents only need to know the present state of the activity; the overall goal, next step or end result is irrelevant for their present work. In Wikipedia, there is no plan specifying which information should be added to the encyclopedia when.
- **memory**: agents do not need to remember their previous activity; no information about the state of the work needs to be stored anywhere except in the medium.
- **communication**: no information needs to be transferred between the agents, except via the work done in the medium; there is in particular no need for the agents to negotiate about who does what.
- **mutual awareness**: each agent works independently; it does not even need to know that others participate. For example, contributors to Wikipedia generally do not know each other or communicate with each other.
- **simultaneous presence**: there is in general no need for the agents to be present at the same time or at the same place; tasks are registered in the medium so that they can be picked up by agents whenever and wherever they are available. That is how worldwide communities can collaborate on a single software project.
- **imposed sequence**: actions are performed automatically in the right order, since an action will not be started until the right condition is in place; the workflow

emerges spontaneously, as the completion of one task triggers the initiation of the next task(s)

- **imposed division of labor**: each agent will only perform the actions for which it has the required competence, i.e. for which it possesses adequate condition-action rules; normally, the more "confident" the agent is about the right action (i.e. the stronger the connection between condition and action), the more it will be stimulated by the condition, and the quicker it will be to start the job; in this way, tasks are automatically assigned to the most competent agents (Heylighen & Vidal, 2008)
- **commitment**: agents do no need to commit to a particular task (in contradiction to what (Jennings, 1993) claims about multi-agent coordination); an agent decides on the spot what work it should do, depending on opportunity and other contingent conditions; an agent that quits or otherwise becomes unavailable is automatically replaced by another one
- **centralized control or supervision**: errors or perturbations are automatically corrected, as they merely create a new condition stimulating new actions to deal with the challenge; the activity is *self-organizing*: global organization emerges from local interactions, without any centralized control directing the activity. For example, bugs in open-source software are spotted by users, and resolved by other contributors.

# Stigmergy as self-organization

This last point deserves a further elaboration. Our assumption is that agents are individually goal-directed. Cybernetics has shown how goal-directedness emerges from negative feedback: perceived deviations from the goal are compensated by counteractions (Heylighen & Joslyn, 2003; Rosenblueth, Wiener, & Bigelow, 1943). This most basic mode of steering is also called *error-controlled regulation*: whatever the origin of the deviation or "error", once it is sensed, its effect is suppressed by an appropriate compensatory action. This control mode does not require any planning, anticipation (feedforward), memory, or understanding of what caused the deviation. To efficiently control the effect, it is sufficient that the agent is able to exert an influence in the "opposite direction" of any deviation, independently of its underlying cause (Gershenson & Heylighen, 2005; de Latil, 1956).

This mechanism is well understood for individual agents (Powers, 1973). Stigmergy illustrates how it can be extended to several interacting agents. Imagine a group of non-communicating agents (e.g. ants, or people who do not speak a common language) pushing a large obstacle out of the way across an irregular terrain. Individually, each agent will correct its course based on the perceived movement of the load: e.g. if it shifts too much to the left, the agent will push more towards the right. It does not matter whether the deviation was caused by a hole in the surface, a sudden gust of wind, or the misdirected action of another agent. The overall movement will be determined by the

sum of the actions performed by all the individuals. As long as the agents push in generally the same direction, it is irrelevant who did what. The agents can work perfectly independently—perhaps even without knowing that someone else is pushing too—while still producing a coordinated movement.

A similar mechanism is probably involved in bodily coordination, where different muscles, joints, tendons and bones contribute to the overall movement of the body. This may be illustrated by the *subsumption* architecture used for the control of the different body parts of many-legged robots (Brooks, 1991), where each limb functions more or less autonomously in helping the robot to move forward, while the only higher-order control imposed is the general direction of movement.

Note that this automatically leads to synchronized action. A complex movement, such as balancing on one foot, requires the simultaneous contraction or extension of many muscles. Each muscle should contract or relax just enough to achieve the intended position. If the actual position deviates from the desired one, each muscle will adjust its level of contraction, upward or downward, depending on the perceived direction of the deviation. Since all muscles have simultaneous access to the shared "trace" (the present position) they also act on it simultaneously, so as to bring it closer to the desired state. The sum of their actions will normally be sufficient to achieve the desired effect. If not—that is, if one of the muscles contributes too little or too much—either that muscle itself or one or more complementary muscles will react in order to correct the error. The muscles do not need to communicate with each other about their intention to shift position: their actions are instantly coordinated by their simultaneous effect on the trace.

To achieve such collective action, the only assumption we need to add to individual error-controlled regulation is that the goals of the agents are not contradictory—i.e. that error decrease for one agent does not equal error increase for another agent—because then the agents will be involved in a tug-of-war of opposing counteractions that will only end when the stronger subdues the weaker. However, note that the goals do not need to be *identical* for coordination to occur: imagine that one group of agents pushes an obstacle to the east, while another group pushes to the north. The net effect is that the load will move northeast, satisfying both groups. It is only when one group pushes eastward and another group westward that a conflict arises, without possibility for a compromise. In this two-dimensional movement example, the probability for conflict still seems large. However, the larger the number of aspects, components or degrees of freedom of the problem situation, the more freedom there is for agents to focus on different goals without getting in each others' way.

This independence of goal setting is what underlies the automatic division of labor: each agent spontaneously focuses on the task that it deems most important (and for which it is in general most competent). Thus, a variety of agents together can potentially tackle very complex problems that require the achievement, in sequence or in parallel, of many different partial objectives. We may assume that agents have acquired their condition-action rules (and thus their implicit goals) through natural selection of instinctual behavior or differential reinforcement of learned behavior. This means that

their condition-action rules are generally appropriate to the local environment, including the other agents with which they regularly interact. Rules that are frequently in conflict with the rules of other agents or the constraints of the environment are likely to be eliminated eventually (Heylighen, 2008a).

Therefore, it is plausible to assume that even very different agents, e.g. belonging to different species in an ecosystem, follow rules that are potentially synergetic (Corning, 1995; Heylighen, 2013b). Stigmergy appears to be a prime mechanism through which this synergy is realized, by coordinating initially independent actions into a harmonious whole. Thus, the group of agents can achieve much more substantial results collectively than they would if they would work alone. *It is this emergence of global order out of local actions that constitutes the hallmark of self-organization* (Heylighen, 2001). It implies in particular that organization arises spontaneously from local activity, without planning, centralization or external control. Problems, contingencies or disturbances will be tackled by the same local action: since there is no plan, there can be no overall deviation from the plan. Any "error" is merely local, and will be corrected by whichever agent is stimulated by that error to perform a compensatory action.

# Affordances, disturbances and feedback

Stigmergy exhibits another fundamental "signature" of self-organization (Heylighen, 2001; Theraulaz & Bonabeau, 1999): positive feedback. Error-controlled regulation assumes negative feedback: the *reduction* of deviations away from the goal. However, goal-directed action can also make use of positive feedback: the amplification of movements towards the goal (Maruyama, 1963). In the traditional cybernetic perspective, changes in the situation not controlled by the agent tend to be interpreted as perturbations, since they move the system away from a previously achieved goal state (Heylighen & Joslyn, 2003; Maturana & Varela, 1980). However, as long as no final goal is reached-which is the default situation in long-term, on-going projects, such as building, extending and maintaining a termite hill-such contingent events may as well facilitate as hinder the further movement toward the goal. When they hinder, we will call them disturbances; when they facilitate, we can call them affordances (Gibson, 1977). In the most general case, we may call them *diversions*, since they divert action from its ongoing course, whether in a positive, negative or neutral way (Heylighen, 2012a). Assuming that agents are implicitly goal-directed, we may infer that they will counteract the disturbances and reinforce or build upon the affordances, i.e. exert a negative, respectively positive, feedback to negative, respectively positive, diversions. Similarly, their reaction to a neutral diversion will be neutral: neither amplifying it nor suppressing it.

Let us illustrate these notions with the paradigmatic case of termites erecting a pillar as part of their nest construction. A bit of mud that is accidentally dropped in a particular place, either by a termite, the wind or a passing bird, constitutes a diversion. In this case, the diversion constitutes an affordance, since it provides a foundation on which

a taller mud structure can be erected. Stigmergic stimulation will lead termites to add mud to the emerging heap, rather than to the flat surfaces surrounding it. The taller the heap grows, the stronger the stimulus it will exert on termites passing by, and therefore the faster its further growth. This positive feedback loop results in an accelerated exploitation of the opportunity, diverting effort away from less promising alternatives, and thus efficiently allocating agents and resources to the most productive activities.

Suppose now that a fragment of the thus erected column breaks off. This constitutes a negative diversion, i.e. a disturbance. In this case, the perception of the missing mud will stimulate the termites to fill the hole with new mud, thus counteracting the deviation from the ideal column shape. Finally, a neutral diversion may arise, such as a breeze blowing some termites off-course, so that they end up near a different column than the one they were heading to. While making the activity deviate from the original course, this event neither facilitates nor hinders the work. Therefore, it will be neither counteracted nor reinforced.

A similar dynamics occurs in Internet communities centered on a particular forum, website or page. More activity on a particular site tends to produce more interesting traces, such as discussions, wiki pages, or comments, which attract more people and therefore more contributions, such as additions to the wiki or replies to proposals, which in turn incite more activity. Vice versa, less activity reduces the level of interest for the products of that activity, and thus the number of potential further contributions. Thus, web communities and their activities are subject to a clear positive feedback, where the initially most promising "projects" grow very quickly, while the less promising ones dwindle, losing the competition with the others, and potentially disappearing altogether. In this way, work is divided relatively efficiently across a wide variety of projects, ensuring that the most promising ones quickly "take off" without dissipating too much energy in less promising ones.

The combination of positive and negative feedbacks is typical for complex, adaptive or self-organizing systems (Heylighen, 2001; J. H. Holland, 1996). It makes the system very flexible, allowing it to act and grow energetically when given the opportunity, while maintaining a stable and robust configuration in the face of disturbances. It also produces differentiation, by amplifying minute differences or chance fluctuations into robust macroscopic structures (Nicolis & Prigogine, 1977). Finally, it makes the system intrinsically non-linear, which implies that for the outside observer its evolution is both unpredictable and uncontrollable.

# Parallel action and the wisdom of crowds

As noted, (stigmergic) coordination has two aspects: *parallel* and *sequential*. Agents or rules working in parallel simply add their effects together. Because their actions are simultaneous, there is no time to *interact*, i.e. for the one to causally affect the other. Therefore, their total effect is simply the aggregate, superposition or sum of their individual effects. Rules working in sequence, however, by definition interact, since the

result of the former affects the performance of the latter. This allows *non-linearity*, i.e. a total effect different from the sum of the individual effects. This total may be larger—in which case there is amplification or positive feedback—, or smaller—which means suppression or negative feedback. As we saw, amplification is useful to exploit affordances, suppression to control disturbances. The combination of parallel and sequential—or linear and non-linear—aspects provides maximal opportunities for an efficient exploration and exploitation of the situation.

Since the power of non-linearity in self-organization is well known, it is worth paying special attention here to the less studied benefits of linear or parallel activity. We can distinguish two cases of parallel action: 1) two or more actions are performed on the same object or task, i.e. the same part or aspect of the medium; 2) actions are performed on separate, independent parts of the medium. The first case may be exemplified by termites adding mud to the same pillar, or agents pushing the same load. The second case can be found when two termites add mud to two different pillars, or when the one is busy repairing the nest while the other is collecting food.

This latter case is perhaps most intuitive, since it underlies the mechanism of the *division of labor*. The advantages of the division of labor are well known: it enables specialization, so that each agent can focus on the task it has most expertise with, and thus the task it can perform most efficiently. We already argued that stigmergy tends to automatically allocate tasks to the most competent agents. To maximally benefit from the division of labor, we moreover need to ensure a sufficient diversity in the competencies of the agents (Martens, 2004): the more diverse their expertise, the more likely it is that at least some agent(s) will be particularly competent for a certain task. As a result, a more diverse group of agents will normally be more productive than an equally large, but more homogeneous, group.

This general principle can be illustrated by a classic ecological experiment: if two identical patches of land are seeded with plants that belong either to one or a few species, or to several different species, the more diverse patch will produce more biomass than the more homogeneous one (Cardinale et al., 2007; Naeem & Li, 1997). (The overall yield increases with the logarithm of the number of species.) The reason appears to be that plants of different species use the available nutrients in somewhat different ways, thus together being able to exploit the resources more completely ("niche complementarity"), while moreover helping each other through synergies (Hector et al., 1999). This is an example of parallel stigmergy where synergetic interaction is mediated by the shared environment (land).

The benefit of diversity is not limited to situations where agents work in different places or perform different tasks. When diverse agents tackle the same problem in parallel, their aggregate solution will in general be better than the one of any single agent or agent type. This phenomenon has been referred to as the "wisdom of crowds" (Surowiecki, 2005) or "collective intelligence". It can be exemplified by the situation where a crowd of people are asked to guess how many beans are contained in a particular jar, or how heavy a particular ox weighs. In such cases, the average of all the guesses is

typically more accurate than any particular guess. The reason is the law of large numbers: if we assume that guesses exhibit a random deviation from the correct answer, then these random deviations tend to cancel each other out when a large number of them are aggregated. Each individual deviation is caused by the limited experience or inaccurate perception of that individual. But when perspectives are diverse, the shortcomings of the ones tend to compensate for the shortcomings of the others, providing a more balanced, and therefore accurate, global perception (Heylighen, 1999, 2013b).

In summary, the greater the variety of agents that work on a particular task or set of tasks, the better we can expect their overall performance to be. Note that parallelism or independent action makes this effect stronger (Surowiecki, 2005): if the agents work in sequence, later ones may still compensate for the limited experience of earlier ones, but because of positive feedback early choices are likely to be amplified. This can lead to the accelerated exploitation of a good solution, but also to the collective converging to a poor solution—a phenomenon termed "groupthink" or "collective stupidity" (Heylighen, 2013b). The development of pheromone trails by ants, which happens partly in parallel, partly in sequence, illustrates the precarious trade-off between the benefits of parallel/linear approaches (more wide-ranging exploration) and those of sequential/non-linear ones (more efficient exploitation) (Heylighen, 1999).

#### Varieties and aspects of stigmergy

Within the broad category of stigmergic mechanisms, we can distinguish many examples and special cases. To bring some order to these phenomena, we will develop a classification of the different varieties of stigmergy. We will do this by defining fundamental dimensions or *aspects*, i.e. independent parameters along which stigmergic systems can vary. The fact that these aspects are continuous ("more or less") rather than dichotomous ("present or absent") may serve to remind us that the domain of stigmergic mechanisms is essentially connected: however different its instances may appear, it is not a collection of distinct classes, but a space of continuous variations on a single theme the stimulation of actions by their prior results.

#### Individual vs. collective stigmergy

Perhaps the most intuitive aspect along which stigmergic systems can vary is the number of agents involved. In the limit, a single agent can coordinate its different actions via stigmergic interaction with its environment.

An elegant example discussed by (Theraulaz & Bonabeau, 1999) is the solitary wasp *Paralastor* sp. building its nest in the shape of a mud funnel. The nest emerges in qualitatively different stages  $S_1, S_2, ..., S_5$ . These subsequently perceived conditions or

stimuli each trigger a fitting action or response:  $S_1 \rightarrow R_1, ..., S_5 \rightarrow R_5$ . Each building action  $R_i$  produces as a result a new condition  $S_{i+1}$  that triggers the next action  $R_{i+1}$ . The wasp does not need to have a plan for building such a nest, or to remember what it already did, since the present stage of the activity is directly visible in the work already realized.

However, the underlying rule structure becomes apparent when the sequence is disturbed so that stages are mixed up. For example, the wasp's initial building activity is triggered by the stimulus  $S_1$ , a spherical hole. When at stage  $S_5$  (almost complete funnel) the observer makes such a hole on top of the funnel, the wasp "forgets" that its work is nearly finished, and starts anew from the first stage, building a second funnel on top of the first one. This little experiment shows that the activity is truly stigmergic, and can only run its course when the medium (the mud) reacts as expected to the different actions performed on it, thus registering the information needed to guide the subsequent actions.

As (Theraulaz & Bonabeau, 1999) suggest, it is likely that collaborative stigmergy evolved from the simpler case of individual stigmergy. Imagine that a second wasp encounters the partially finished nest of the first wasp. It too will be stimulated to act by the perception of the present state of work. It does not matter that this state was achieved by another individual: the wasp anyway has no memory of previous actions—its own or someone else's. Assume further that the resulting structure is big enough to house the two wasps. In this case, the wasps will have collaboratively built a nest for both, without need for any additional coordination between their genetically programmed building instructions. Assume that the structure is modular, like the nests of social wasps, so that an unlimited number of modules can be added. In that case, the number of wasps that may start working together simply by joining the on-going activity on an existing nest can grow without limit.

This example illustrates how the number of agents collaborating on a stigmergic project is actually much less fundamental than it may seem. The essence of the activity is always the same. Assuming that the agents have the same competencies, adding more agents merely increases manpower and therefore the size of the problem that can be tackled, the speed of advance, or the eventual magnitude of the achievement. Only when the agents are diverse can an increase in their number produce a qualitative improvement in the solution.

The only complication added is that agents may get in each other's way, in the sense that similar individuals perceiving the same stimulus are likely to move to the same place at the same time, thus obstructing each other's actions. This problem is easily tackled by an additional rule, which is already implicit in individual work but likely to become reinforced during collaborative work: *keep a minimum distance from obstacles*—including other agents. This rule is a well-known ingredient in the many successful simulations of collectively moving animals, such as flocks, schools or swarms (Okubo, 1986), allowing densely packed groups of agents to follow complex, synchronized trajectories without ever bumping into each other. In combination with the basic stimulation by the stimulus object, this leads to what may look like a carefully thought-

out strategy of coordinated movement. An example are group hunting strategies, as used e.g. by lions or wolves (Parunak, 2006). Each wolf is attracted to move towards the prey (basic stimulus). On the other hand, each wolf is stimulated to stay as far away as possible from the other wolves. The result is an efficient encirclement of the prey, which is attacked simultaneously from all sides with no opening left for escape.

# Quantitative vs. qualitative stigmergy

Quantitative stigmergy (Theraulaz & Bonabeau, 1999) refers to perceived conditions that differ in strength or degree, and where stronger traces typically elicit more forceful (intense, frequent, ...) actions. This quantitative variation is perhaps best captured using my definition of stimulation in terms of conditional probability: the stronger the trace, the larger the probability of a certain action given that trace. Over an extended period, higher probability implies more frequent actions by more numerous agents, and therefore more intense overall activity. The two paradigmatic cases of stigmergy, termite nest-building and ant trail-laying, follow this quantitative logic. The higher the emerging heap of mud (stronger trace), the more an individual termite is attracted to it, and therefore the larger the probability or frequency of mud being added. The stronger the scent of pheromone on a trail, the less likely an ant is to deviate from that trail, and therefore the higher the probability that it too will reinforce the trail with additional pheromone. These are typical examples of the positive feedback that efficiently amplifies positive developments.

But quantitative stigmergy can also be exemplified by negative feedback, where a stronger trace leads to less activity. A human example can be found in the market mechanism. Extensive buying of a good (action) reduces the supply and thus increases the price, which is a quantitative trace left by the collective buying and selling activity. A higher price will normally reduce the probability that someone would buy additional stock of that good (negative stimulation). Thus, a higher price reduces demand, which in turn will reduce the price. This mechanism of self-organizing, distributed control (Heylighen, 1997) implements the "invisible hand" of the market. It stabilizes prices and efficiently allocates production capacity to the goods that are most in demand (Witt, 2006).

Qualitative stigmergy (Theraulaz & Bonabeau, 1999) refers to conditions and actions that differ in kind rather than in degree. In this case, a different trace stimulates a different type of action. An example can be found in the different stages of the building of a funnel-shaped nest by the solitary wasp that we discussed, where each stage requires a particular type of building action. A human example can be found in "wiki" websites that are edited by their own readers. A paragraph that contains a semantic mistake (e.g. in the definition of a word) will elicit a corrective action (e.g. writing a new definition). Different types of errors, vagueness, or lack of information will stimulate different types of additions and corrections.

In practice, there is no clear boundary between quantitative and qualitative cases of stigmergy. All non-trivial activities require a choice from a range of potential actions. Which of the different possibilities will be chosen is typically determined probabilistically: in some conditions one type of action is more likely, in other conditions another type of action. As the one condition becomes more similar to the other, the probabilities become more similar too. In the middle, the two probabilities may become equal, as in the situation of Buridan's ass, which had to choose between two equally attractive options. More generally, we may assume that the probability is equal to  $P(a_i | c)$ , where  $\{a_i | i = 1, ..., n\}$  is a discrete set of possible actions, while the condition  $c \in C$  varies continuously over the space C of all states that the world can have. In this model, the probability (and therefore frequency or intensity) of an action varies approximately continuously (quantitative variation), while the action itself is chosen from a discrete range of options (qualitative variation).

#### Sematectonic vs. marker-based stigmergy

Grassé's original definition of stigmergy concerned stimulation by the performed work itself: in his observation, termites are stimulated by the mud heaps they have already built. E. O. Wilson (1975), in his monumental "Sociobiology", called this stimulation sematectonic. However, in many cases social insects appear to be stimulated by pheromone traces, which are left expressly as a signal for others, not as a contribution to the work itself. In fact, it later turned out that termites are actually stimulated by the pheromones mixed in with the mud by co-workers rather than by the mud itself (which was to be expected given that termites are blind). The situation is even clearer for ants laying trails. In principle, ants could be guided by the perceivable results of their activity-the way humans and large animals are guided by the trails of flattened vegetation and sand eroded by the movement of previously passing individuals. However, the effect of an ant's movement on its surrounding is so small as to make it practically undetectable. Therefore, ants appear to have evolved a special type of chemical markers-pheromones-that make the traces of their activity much more salient. This type of indirect stimulation, not by the work itself but by a specially evolved "side-effect", has been called marker-based stigmergy (Parunak, 2006).

The evolution of markers is an obvious method to make stigmergy more efficient, by more reliably focusing the agents' attention on the most relevant aspects of the work that needs to be done. However, it entails an extra cost and complication in that individuals need to perform the task of manufacturing markers in addition to the work itself. A human example can be found in the Wikipedia encyclopedia on the web. Readers are stimulated to improve existing pages either directly, by reading the text and noticing its shortcomings, or indirectly, by reading comments that summarize the tasks that still need to be done—such as adding references, clarifying ambiguous sections, or checking facts (Heylighen, 2007). The direct method exemplifies sematectonic

stigmergy, the indirect one marker-based stigmergy. The "markers" in this case are the various "to do" notes that attract the attention to the problems that still require work.

A marker can be seen as an abstract, conventional sign, intentionally representing the work to be done instead of mechanically registering its effects. In Peirce's semiotic taxonomy of signs (Atkin, 2010; Burks, 1949), a marker is a *symbol*, while a sematectonic trace is an *index*. As such, a marker may seem to belong to a higher-order semiotic or communicative category of phenomena—a "meta-level" compared to the "object level" of the work itself. However, as in all phenomena produced by evolution, there is an essential continuity between the more primitive and the more "advanced" versions, as we can illustrate with a well-known example.

Many animals mark their territory by leaving traces of urine all around it. Obviously, excreting urine was not initially intended as a communicative signal, but merely as a way to get rid of liquid waste products. But since urine is easily perceived because of its smell, while its presence is causally connected to the presence of its producer, animals quickly learned to interpret it as a sign ("index") of the presence of another animal in the vicinity. Such a signal constitutes possibly vital information, which is useful, both for the receiver, who is warned of a potentially dangerous rival, and for the emitter, who can use it to frighten away newcomers from its territory. Thus, both parties are taught by evolution to communicate more reliably by means of this signal, turning it into a conventional marker of territory. As a result, animals have learned to deposit a little urine at regular intervals around their territory rather than simply emptying their bladder in a random place when it is full. This marker now supports stigmergic coordination between foraging activities, by clearly delimiting each individual's hunting grounds, and thus minimizing the risks of encounters ending in conflict.

The effect is equivalent to the human institution of "property rights"—the formal establishment of what belongs to whom, which economists consider essential for dependable transactions (Martens, 2004). The simplest way to establish a property right is to put a fence around the territory that you consider to be your property. Like the urine trace this provides a clearly perceivable signal to others that they should not trespass there, obviating the need for individual communication with each of those others.

In the animal case, we see how a phenomenon (smell) that was merely a side effect of a primary action (getting rid of waste products) turned into an intentional, communicative signal—even though its primary function of waste disposal is still essential. In the case of pheromones, this original function, whatever it may have been, seems to have been lost, leaving only the communicative function. But in the most general case, both functions, primary and communicative, are likely to play a part. The fence, for example, not only warns people not to trespass, but keeps cattle from getting out. Another human example is an artist making a sketch. The sketch functions both as a first step towards performing the intended work (e.g. drawing someone's portrait) and as a representation of what the finished work may look like—which can be used to convince a sponsor who may be interested to order the finished work. The first function is sematectonic, the second one marker-based.

#### Transient vs. persistent traces

After discussing basic aspects of stigmergy that are recognized in the literature (e.g. Parunak, 2006), I wish to suggest a new dimension of variation. Parunak, in his attempt at classification, proposed the dynamics of the environment (what I call medium) as a crucial factor in stigmergy. However, there exists an infinite variety of potential dynamics of different degrees of complexity, thus making classification practically impossible. Moreover, a non-trivial dynamics seems better captured by causal rules, and as such by a system of (agentless) actions transforming the state of the world. For example, a collectively edited website, like Wikipedia, may have some in-built procedures that automatically correct formatting errors, add links, or signal incoherencies. The fact that these actions are performed by computer programs (e.g. "bots") does not fundamentally distinguish them from the actions of human contributors, since they all undergo the same stigmergic coordination. We have conceptualized the medium as the *passive* component of the stigmergic system, which undergoes shaping by the actions, but does not participate in the activity itself.

But even a passive medium is subjected to dissipation, erosion, or the increase of entropy entailed by the second law of thermodynamics. That means that structures and markers tend to decay spontaneously—unless they are actively maintained and reconstructed. Examples are the evaporation of pheromones and the wearing down of termite hills by rain, wind and gravity. This decay is not a priori negative. The traces left in the medium function as instructions for further work. It is obvious that without continuing updates this information will little by little become obsolete as the situation changes. For example, pheromone trails that point to exhausted food sources have become not just irrelevant, but misleading, since they incite ants to make useless journeys. Happily, pheromone trails that are no longer reinforced—because ants following them do not return with food—will gradually diffuse, and thus lose their attractiveness relative to trails that continue to receive reinforcement.

This is the same phenomenon of selective "forgetting" that characterizes memory in the brain: neural connections that are no longer reinforced will gradually lose their strength relative to recently reinforced ones. The speed of this forgetting depends on the *learning parameter*, as defined in neural networks (Heskes & Kappen, 1992). A large value of the parameter means that new changes in connection strength are large relative to the cumulative effect of previous ones, thus promoting the speedy establishment of new memory traces—but also the quick obsolescence of older traces. A small value, on the other hand, means that older learning episodes continue to exert a strong effect. A similar parameter probably controls the external memory of ants as laid down in pheromone trails: newly added pheromone should be strong enough to allow trails towards newly found food sources to eventually become more attractive than previously found ones; yet, it should not be so strong that some recent journeys by ants carrying

food from a new, unproven source can overpower the signals pointing to an older source whose reliability is evidenced by hundreds of successful journeys (Heylighen, 1999).

Given that what counts is the relative attractiveness of different options for action, the "learning" parameter, which determines the intensity of new contributions to the trace, is in practice equivalent to a "forgetting" parameter, which determines the rate of decay of the existing trace. The optimal value of this parameter will depend on the speed with which information becomes obsolete. This will depend on the variability in the environmental diversions and the measures that are taken to control them. For example, the location of a particular pillar in a termite hill is unlikely to become obsolete quickly, since the disturbances and affordances that it regulates, such as protection against sun, cold and predators or the creation of a comfortable interior microclimate, generally do not change position. Abundant food sources for ants, on the other hand, tend to change location every few days or hours.

Some diversions, such as the sudden appearance of a predator or prey animal, are even more short-lived. In this case, a trace inciting the appropriate action should be as quick to appear as to disappear. Typical stigmergic signals will be acoustic (e.g. the warning cry uttered by a monkey that spots a snake—which is marker-based) or visual (e.g. the visible movement of a wolf towards a deer—which is sematectonic). The reason is that sound and light, because of their wave nature, spread and decay almost immediately. An intermediate decay speed is typical for chemical traces in a liquid environment, where concentrations of molecules may change within minutes. An example of such kind of stigmergic coordination are the chemical signals broadcasted by bacteria that encounter either an affordance, such as food, or a disturbance, such as a concentration of toxins (Ben-Jacob, Becker, Shapira, & Levine, 2004). The first type of diffusing signal will create a chemical gradient that incites bacteria of the same colony to swim towards the food source, so that they too can profit from it. In the second case, the gradient will incite them to move away from the danger threatening their congener.

These examples illustrate once again that no sharp distinction can be made between *persistent* and *transient* traces used in stigmergy: these are merely the opposite ends of a continuum. Yet, the distinction may be useful for conceptual clarification. Persistent traces lead to what may be called *asynchronous* stigmergy: the different agents or productions do not need to be present at the same time, since the trace remains to guide them at any later time. Asynchronous communication (Cristian, 1996) can be illustrated by media such as print, email, or websites. Its advantage is that information remains available, so that it can be processed at the most appropriate occasion, and can accumulate and mature over the longer term. Transient traces lead to *synchronous* stigmergy: the agents need to be simultaneously present for the coordination to succeed. Synchronous communication may be exemplified by media such as telephone and Internet "chat". Its advantage is that interaction, and therefore feedback, is instantaneous, so that disturbances and coordination errors can be corrected without delay.

Synchronous communication is rarely conceived as stigmergic, since it is typically used for direct interaction, such as conversation or discussion. Yet, a warning

cry or a chemical signal exemplify indirect interaction: they are targeted at no one in particular but merely "released" in the medium. Examples of stigmergy in synchronous interaction are even clearer when the signal is sematectonic. For example, a bird spotting a danger (condition) will start to fly (action), and by this example (transient trace) set off the whole flock to fly away (subsequent action). Synchronous stigmergy may be best exemplified by the collective movement in herds, flocks or swarms (Moussaid, Garnier, Theraulaz, & Helbing, 2009; Okubo, 1986), where the agents are continually adjusting their trajectory on the basis of real-time perceptions of the movements of other agents.

A human example would be the self-organization of traffic, where drivers continuously react to the traffic conditions they perceive, by e.g. stopping, accelerating, or changing lanes, thus affecting these very conditions and the subsequent actions of other drivers. Roads, lanes, road markings and traffic signs, on the other hand, function like a persistent trace developed over decades in order to stimulate the drivers to move in a coordinated manner. The continuity between the two is demonstrated by the fact that in sufficiently dense traffic lanes tend to self-organize and acquire some form of stability, even when they leave no permanent trace (Helbing, 2001; Moussaid et al., 2009). Nevertheless, when the surface is soft enough to show signs of erosion, like in dirt roads, traces persist after the traffic stops, thus maintaining a memory of the self-organized traffic pattern. This persistent trace reduces the time necessary to rebuild a coordinated movement pattern when the traffic starts up again. It seems likely that most roads have emerged in this manner across historical time.

#### Broadcast vs. Narrowcast

Another basic component of the stigmergic taxonomy proposed by (Parunak, 2006) is the topology of the medium (or "environment"). Here the same difficulties arise as with the dynamics: the potential topologies are unlimited in number and complication, making classification intrinsically hard. Again, I suggest replacing this multidimensional, qualitative notion by a one-dimensional, quantitative aspect: the range or *scope* of the stigmergic process. The scope represents the size of the "neighborhood" across which a stigmergic signal is perceivable. The two poles of the scope continuum may be called *broadcast* and *narrowcast*. Broadcasted traces can be perceived by all agents involved. Narrowcasted traces are perceivable by only one or a few agents. This will obviously depend on the topology of the medium: a large trace in an uninterrupted, flat plain will be visible from afar; the same trace in a landscape cut through by rocks, valleys and trees will only be visible in a small region. It will also depend on the degree of diffusion of the traces that remain localized, such as shapes and inscriptions.

As yet, there does not seem to be much research to clarify the differences between broadcast and narrowcast. Implicitly, most studies of stigmergy assume broadcast within a given group of agents, such as an ant colony, or an open source community. But

obviously, these groups themselves are limited in scope, and therefore there is always a degree of narrowcast.

The situation becomes more complex—but also more interesting—when different actions or agents have a different scope, so that A's traces e.g. may reach B, C and D, while D's traces reach B and E. In this case, the topology of the stigmergic medium becomes equivalent to a *network* where different nodes (A, B, C...) each are connected to (i.e. can deposit traces perceivable by) different other nodes. The implication is that the network paradigm—which is increasingly popular for modeling various complex and self-organizing systems such as neural networks, social networks, citation networks, etc. (Heylighen, 2008b; Newman, 2003)—could be viewed as a special case of the stigmergic paradigm, albeit a rather complicated one. The stigmergic paradigm remains more general than the network paradigm in the sense that the scope of a stigmergic interaction can vary, while the "scope" of a network connection is fixed. For example, a more intense trace (e.g. more concentrated pheromone) will typically spread over a somewhat larger scope, and thus influence more agents.

This stigmergic perspective may actually clarify some problems in traditional network models. For example, we know that in the brain connections are not fixed, since neurons can grow axons to connect with remote other neurons. To guide this growth pattern, some neurotransmitter-like signal molecules must be able to diffuse outside of the existing neurons and synapses, implying a more "broadcast" form of stigmergic communication. Similarly, social networks are everything but well defined and fixed in their scope: people's actions will typically have repercussions well beyond their present friends and acquaintances, potentially bringing them in contact with a much wider circle of people. We will leave these issues for future work, and just note that a stigmergic analysis may extend even to typical network models, such as connectionist theories of learning and thinking in the brain (see also (Heylighen, 2012b)).

# Extending the human mind

Now that we have surveyed the most general properties of stigmergy in human, animal and physical systems, it is worth investigating some specific applications to human behavior. Next to its general function of coordination, stigmergy in particular supports cognition and cooperation.

Traditionally, cognition has been viewed as the processing of information inside the brain. More recent approaches, however, note that both the information and the processing often reside in the outside world (Clark, 1998; Dror & Harnad, 2008; Hollan, Hutchins, & Kirsh, 2000)—or what we have called the *medium*. For example, documents function as an external memory for storing knowledge and data, while calculations are typically performed on a piece of paper or on a calculator. Without such supporting media, most advanced reasoning—as performed e.g. in science and technology—would be simply impossible. Thus, the human mind *extends* into the environment (Clark &

Chalmers, 1998), "outsourcing" some of its functions to external support systems. The reason is that our memory and information processing capabilities have rather strict limitations (Heylighen, 2013a; Heylighen & Vidal, 2008)—most famously the "magical number 7 plus or minus 2" which denotes the maximum number of items we can hold in short-term memory. Books and computers are relatively recent inventions. However, the use of an external medium for supporting cognition is probably as old as cognition itself.

More precisely, our mental capabilities can be seen as an interiorization of what were initially stigmergic interactions with the environment. The perspective of situated and embodied cognition (Aydede & Robbins, 2009; Steels & Brooks, 1995) focuses on the interaction between the agent and its environment: the agent senses the state of the environment via its sensory organs and reacts to it by producing an appropriate action via its muscles or effectors: it reacts in the same way to the returning feedback signal. Such a reaction requires merely a condition-action rule, which, as we saw, is nothing more than a causal process transforming an antecedent into a consequent. As such, condition-action rules are readily implemented in the simplest systems, such as thermostats. Yet, when the activity of these rules is coordinated by stigmergy, it becomes capable of complex, goaldirected behavior, such as building a wasp's nest, or a spider's web.

A classic example of the "intelligence" exhibited by such simple rules can be found in Braitenberg vehicles—rudimentary automata equipped with just two sensors (left and right) for light intensity, two wheels for movement, and connections between them that increase the speed of the wheel in proportion to the intensity of light perceived by its corresponding sensor (Braitenberg, 1986; Gershenson, 2004). The effect of these causal connections is that the vehicle follows a complex trajectory that avoids as much as possible all light sources, while apparently seeking a place of darkness where it comes to rest. The very efficient adaptive and goal-seeking behavior of a Braitenberg vehicle results from the stigmergic coordination between the actions performed in sequence or in parallel by its wheels when reacting to sensed changes in conditions, where the one complements or corrects the results produced by the other (Heylighen, 2010).

Both the strength and weakness of such stigmergic organization is that it lacks internal memory: information about the state of the process is stored purely in the medium from where it is sensed by the agent. The advantage is that there is no need for the registration, maintenance, and recollection of information in the brain. The disadvantage is that if the medium is disturbed, the trace and with it the memory may be erased. We saw an example of this problem with the nest-building wasp: when the experimenter creates a misleading trace on the nearly finished nest, the wasp starts building a new nest on top of the old one, thus uselessly duplicating its effort. It is likely that our capability for internal information storage evolved at least in part to avoid this problem: if the state of the activity can be registered and processed internally, complex activities can be planned even when the external medium does not cooperate.

Thanks to this capability, humans are much smarter than insects. Nevertheless, our brain is an energy-intensive, costly organ, whose storage capacities remain quite limited. That is why we continue to use stigmergy to support our memory and reasoning.

Let us discuss a few examples. Whenever we have to do a complex job, such as repairing a bicycle, preparing a dinner, or filling out our tax forms, we tend to keep both the objects we work on, and the different tools and resources that support the work at hand, in such a way that they are easy to see and to use.

For example, while taking apart the bicycle we arrange all the screws and pieces in clear view, close to the screwdrivers or pincers we will need to put them back on, so that we are unlikely to forget what must be added when and where. Each tool or piece is a stimulus for performing a particular action. The perceived state of the bicycle is the condition that determines which action is to be performed when. If before we start we had to analyze, plan and memorize all the steps that need to be taken in dissembling, repairing, and then reassembling the bicycle, it is unlikely that we would ever succeed in this task. The arrangement of the physical components in space here plays the role of the activity's trace, which constantly guides the stigmergic coordination of actions.

Ergonomic studies have shown that the spatial arrangement of a workplace is crucial to the efficient performance of work (Hollan et al., 2000; Kirsh, 1995, 1996). One obvious reason is that when tools are positioned near to where they are likely to be used, there will be less need for physical movement. However, stigmergy reminds us that good arrangement saves cognitive effort as well as physical effort, by connecting the right reminders to the right circumstances. For example, one of the reasons why "Post it" notes are so popular is that they make it easy to spatially connect a cognitive "call for action" (challenge, stimulus, marker) with the physical resource needed to perform the action. Sticking a "Please photocopy!" note on a document, e.g., makes it obvious for anyone passing what needs to be passed through the copying machine.

The full power of individual stigmergy is seen with creative work—such as drawing a picture, writing a text, or modeling a piece of clay. Here, the provisional results of the work are fully embodied in the trace, be it a sketch, a draft document, or a clay shape. This preliminary registry of the work performed calls out for more. It challenges the user to add, to enhance or to correct. Each addition changes the trace, thus attracting the attention to further imperfections, or suggesting further additions. It would be extremely difficult, if not impossible, to achieve the same level of sophistication in a design that would only exist inside the creator's brain, where all the planning would take place without any external medium to store it, test it, and be challenged by it.

While painters or writers may have a general idea of the piece they want to create, the actual details will only take shape when that idea is exteriorized in a medium that can be scrutinized and manipulated, so that its structure step-by-step acquires the ideal shape for the purpose. That makes it possible to take into account all the possibly unforeseen properties and side effects of an initially still abstract idea. This principle is at the basis of the method of *stigmergic prototyping* (De Couvreur, Detand, Dejonghe, & Goossens, 2012; Dejonghe, Detand, & De Couvreur, 2011), in which a conceived artefact is immediately given a rudimentary physical shape that can be easily tested out and thus adapted to the circumstances. In contrast, the traditional approach first tries to design a detailed, abstract blueprint of the artefact, but then often has to conclude that its physical

implementation does not work as intended, forcing the designer to go "back to the drawing board".

# The evolution of cooperation

As we have noted several times, stigmergy does not distinguish between individual and collective activity: the trace left in the medium coordinates actions, while being indifferent as to the specific agent or agents initiating these actions. The only additional requirement for collective action is that the different agents should not work at cross-purposes, so that the one's actions negate or obstruct the other one's. But even such a conflict tends to remain localized to a small part or aspect of the trace, while allowing the rest of the trace to develop unhindered.

An example can be found in Wikipedia "edit wars" (Sumi, Yasseri, Rung, Kornai, & Kertész, 2011), in which two contributors who disagree about a particular statement in a Wikipedia article repeatedly undo each others' corrections. This does not prevent other contributors from elaborating the rest of the article (and the encyclopedia). Often, the conflict tends to get resolved by a third party who proposes a compromise statement that the conflicting parties no longer object to. Even without third party intervention, the conflict is unlikely to continue, either because the antagonists themselves chance upon a statement that is acceptable to both, or because one of them simply gives up repeating the same ineffectual action, and decides to focus on some more productive task.

From this stigmergic perspective, the emergence of cooperation between selfish individuals seems a much less daunting issue than from a traditional evolutionary or economic perspective (Axelrod, 1997). Traditional models of the evolution of cooperation pit one individual against another one in a Prisoners' Dilemma type of interaction, where it pays to "defect" (i.e. be uncooperative) in the short run, even though everybody would be better off being cooperative in the long run. Another common paradigm is the Tragedy of the Commons, in which selfish individuals ("free riders") exploit—and eventually exhaust—the common good that others try to maintain cooperatively (Feeny, Berkes, McCay, & Acheson, 1990; Hardin, 1968). For example, a person who consumes more than his fair share of a common resource, such as water, grass, or land, will leave less of the resource for the people dividing up the resources more evenly. In such cases, the cooperative arrangement tends to be undermined by selfish agents appropriating more benefit from it than earnest cooperators, thus tempting others away from cooperation.

In the stigmergic paradigm, the common good (e.g. Wikipedia, or a network of trails and roads connecting common destinations) is gradually built up via the cooperation implicit in stigmergically coordinated actions. Free riders may profit from this common good without putting in any effort in return. However, the benefit derived from a stigmergic trace does not in general reduce the value of that trace. For example, an ant that follows a pheromone trace laid by others without adding pheromone of its own does not by that action make the pheromone trace less useful to the other ants. Similarly,

a person who downloads a piece of open source software without contributing to the development of that software does not impose any burden on the software developers. Thus, in a situation of stigmergy, a free rider or "defector" does not weaken the cooperators, in contrast to situations like the Prisoners' dilemma or Tragedy of the Commons.

In a sense, by not contributing the free riding agent merely weakens its own position, because it passes by the opportunity to adapt the trace to its own preferences. As we saw, the stigmergic trace is the aggregate of many independent actions, each of which helps the agent that performed it to achieve its goals. The ant that finds food but does not leave a pheromone trace on its way back to the nest not only does not help others to get to that food: it also does not help itself, because without the trace it is very unlikely to find the same food source again. The trace is both an individual and a collective "mental map" that indicates effective actions (Heylighen, 1999). Not leaving a trace makes your own future work harder than it needs to be.

Let us analyze the dynamics of free riding in more depth on an example inspired by what may be the simplest type of stigmergy, the creation of a trail across irregular terrain through the flattening of grass, dirt and other obstacles. Here, an easy-to-travel path emerges as a side effect of the regular movement of people or animals, while requiring no special effort from these agents. A more demanding version of this task is the establishment and maintenance of a path through dense vegetation, like in a forest. As quickly growing bushes and trees extend their branches, they eventually obstruct the path. A person following that path will have to either duck around these obstacles, or remove them, e.g. by cutting the twigs that intrude upon the open space. The first option may demand somewhat less effort, but that applies only to the short term, as the underbrush will grow until it becomes impassable. Somebody who regularly uses the path will be motivated to follow the second course of action, and remove any obstruction before it becomes insurmountable.

This preference is independent of the number of hikers actually using the path. Yet, the larger the number of people applying the strategy, the less work any one of them needs to perform. Thus, their actions are cooperative, as they help each other achieve their objectives. But such cooperation is purely stigmergic, because they travel independently of each other, at different times, and thus do not communicate about their common purpose. People who only use the path occasionally may not contribute to this ongoing clearing activity, and thus "free ride" on the effort of others. But unless the others do enough work, the ones who use the path regularly will eventually have to make the effort for purely selfish purposes, because without that effort they will not be able to use the path anymore.

This example resembles the Prisoners' Dilemma or the Tragedy of the Commons in that there is a temptation to defect by letting the others do the hard work, while profiting from their results. The crucial difference, however, is that such a free riding strategy will eventually hurt the defector more than the "cooperator", because the cooperating agent will continue to clear its own path independently of any others

(cooperators or defectors) using that path, while the defecting agent will eventually encounter a path that has become impassable without clearing effort, forcing it to either become a stigmergic cooperator, or give up on passing altogether. Thus, a defector will in the long run collect less benefit than a cooperator. This makes the strategy of noncooperation self-defeating.

In the short run, the free rider may seem to have the benefit over the cooperator of spending less energy establishing and maintaining the trace. However, the cooperator collects other benefits. First, as we noted with the ant leaving pheromone or the hiker breaking off branches, the cooperating agent helps itself by creating a trace. Second, the stigmergic interaction will boost the benefits of that individual trace by stimulating others to expand on it. For example, an ant creating a trail to a new food source will incite others to explore the neighborhood of that source, potentially discovering even better sources or shorter trails. Similarly, the hiker who partially cleared a path will thus increase the probability of others following that same path while performing further clearing themselves. This is the positive feedback of actions eliciting more actions that makes stigmergy so effective. The free rider simply misses out on this potential amplification of its actions.

The full power of such synergetic interaction supported by stigmergy is seen in complex, creative work environments, where different agents contribute different skills, experiences and perspectives. Here, the work done by one individual is enhanced by the work of others with complementary abilities in a way that the single individual never could have achieved. Wikipedia and communities developing open source software development are prime examples, having achieved results that could not even have been reached via hierarchical, command-and-control strategies of coordination (Heylighen, 2007; Heylighen et al., 2013). Smaller scale examples are people posting photos, ideas, artwork, or essays on their blog, Twitter feed, or Facebook page, and getting feedback from friends, followers, or strangers, which help them to further develop their insights, while inspiring these others to build further on their experiences. In such cases, the benefits that accrue to the "cooperators" are direct, concrete, and stimulating enough to motivate them to produce more of such "public traces" in their medium of choice (Wikipedia, Facebook, ...).

Thanks to the user-friendly electronic medium, the material and human cost of publishing such traces is nearly zero. This combination of strong motivation, minimal cost, and effective stigmergic coordination turns the medium into a powerful system for mobilizing joint action (Heylighen et al., 2013). The result is a rapidly expanding "collaborative commons" (Rifkin, 2014)—a virtual workspace for stigmergic (and more traditional) cooperation that encompasses the planet. This world-wide stigmergic medium is presently developing into the equivalent of a *global brain* able to efficiently tackle the collective challenges of society (Heylighen, 2008a, 2014).

While the ICT applications of human stigmergy most stir the imagination because of their virtually unlimited scale, we need to remember that the same mechanism has been supporting collaboration across human and evolutionary history. A final example

may illustrate some of the more down-to-earth applications. People who garden like to show off the fruits of their labor to visitors, guiding them along flower beds, vegetable yards and fruit trees. Visitors with some knowledge of gardening will spontaneously comment on what they see. The resulting exchange of knowledge is triggered by the visible trace of the gardening work, in which visitors e.g. note that certain flowers in the garden are doing better or worse than in their own garden, prompting them to either ask or give advice on how to tend that particular variety. If the garden was communal, this sharing of information would naturally extend into sharing of physical work on the garden, with individual gardeners concentrating on the plants or tasks they feel most competent about or that are most in need of work. Thus, they create a more beautiful garden for all, while reducing the individual workload.

Most forms of human cooperation have this stigmergic dimension, where actions are triggered by the observable results of other people's actions rather than by direct requests or commands. The traffic example may remind us that most people do not require the directions from a policeman in order to cooperatively produce a smooth flow of vehicles. But because the explicit request from a policeman, co-worker or boss to perform a particular action requires conscious processing—if only to decide whether we will honor the request or not—we tend to be much more aware of such direct communication. Therefore, it tends to remain in our memory as a driver of our actions. Our reactions to the implicit challenge left in an evolving piece of work, on the other hand, tend to be subconscious and automatic. Therefore, we assume that we decide to perform a further action purely on our own initiative, ignoring that we are actually being driven by the stigmergic organization of the medium. But the coordinated activity that ensues is a truly ubiquitous mode of human cooperation, albeit one that has hardly received any attention until now.

# Conclusion

Our theoretical analysis and survey of the mechanism of stigmergy has illustrated how wide-ranging, concrete and fundamental its applications are. Virtually all evolved processes that require coordination between actions rely at some level on stigmergy, in the sense that subsequent actions are stimulated by the trace left by previous actions in some observable and manipulable medium. The trace functions like a registry and map, indicating which actions have been performed and which still need to be performed. It is shared by all agents that have access to the medium, thus allowing them to coordinate their actions without need for agent-to-agent communication. It also allows individual agents to perform complex sequences of actions without need for a memory or plan that keeps track of which action needs to be performed when. It even allows the coordination of "agentless" actions, as exemplified by chemical or physical reactions, and investigated by Chemical Organization Theory (Dittrich & Fenizio, 2007)—a promising new approach for modeling the emergence of self-sustaining systems.

Thus, stigmergy can be seen as a fundamental mechanism of *self-organization*: it allows global, coordinated activity to emerge out of local, independent actions. Like self-organization in general, stigmergy relies on *feedback*: action elicits action, via the intermediary of the trace. This feedback is typically positive, in that actions intensify and elaborate the trace, thus eliciting more intense and diverse further actions. The resulting virtuous cycle explains in part why stigmergic organization is so surprisingly effective, enabling the construction of complex structures—such as a termite hill, a network of trails, or a world encyclopedia—in a very short time, even when starting from scratch. When necessary, feedback can also be negative: errors, disturbances or "overshoots" that make the trace deviate from its ideal shape will elicit actions that correct the deviation.

We have examined different variations on this theme by distinguishing basic aspects or dimensions in which the process can vary. The number of agents involved turns out to be less fundamental than is generally assumed. Increasing that number will qualitatively enhance the result only if the agents are sufficiently diverse in the actions they contribute, yet sufficiently aligned in their strategies so that they do not hinder each other. Like the number of agents, the difference between qualitative and quantitative stigmergy does not seem essential, given that the notion of "stimulation" entails a quantitative aspect of intensity or probability, while the actions that are stimulated more or less intensively differ qualitatively. The difference between sematectonic traces-the concrete, observable results of work performed-and markers-traces left to guide subsequent actions, but without contributing to the work itself—is important but subtle. The use of markers allows a more fine-grained control of stigmergic coordination, but demands an advanced level of collective evolution, in which certain traces have acquired a conventional meaning among the agents that use them. The *transience* of the trace is crucial in order to ensure that the list of "to do's" remains up to date: in a quickly changing environment, actions need to adapt in time to new circumstances, which means that an outdated trace should decay before it would elicit too much useless activity; in a more stable environment, on the other hand, persistent traces enable the accumulation of a long and detailed memory. A final dimension of variation, the broadcast-narrowcast continuum, is as yet insufficiently investigated. Nevertheless, it hints at an essential continuity between stigmergic mechanisms of self-organization (using a typically broadcasted trace) and the better-known, local interaction mechanisms, where the result of an action only affects linked or neighboring agents.

We have then examined two more typically human applications of stigmergy, the support of cognition and cooperation, while emphasizing their evolution out of more primitive mechanisms that can be traced back to the first living organisms. *Cognition* is an application of individual stigmergy: the trace of activity in a medium functions like an external memory that facilitates storage and processing of information, thus reducing the burden on the brain. However, the fact that this mechanism supports some degree of intelligent activity even without a brain suggests that our cognitive abilities may have evolved by simply interiorizing some of this functionality that was initially provided by an external medium.

*Cooperation* is an effect of collective stigmergy. Stigmergic coordination arises spontaneously, without need for any cooperative intent from the individuals. Since coordination is beneficial to the agents involved, evolution is likely to strengthen the condition-action rules that make them interact synergetically, while eliminating the rules that produce conflict or friction. Stigmergy moreover bypasses a classic obstacle to the evolution of cooperation: the "tragedy of the commons" where "free riders"—who profit from the fruits of cooperation without contributing to it—do better than the cooperators, therefore eventually outcompeting them and thus destroying the cooperative arrangement. The free rider problem is avoided because cooperators (1) do not lose any benefit, since the trace typically does not deteriorate through free rider exploitation; (2) get the additional benefit that the work they do not only helps themselves, but stimulates others to expand on it. Therefore, the free rider benefit of avoiding effort in building the trace does not seem large enough to allow them to outcompete the cooperators. At worst, free riders may continue to live side-by-side with cooperators, each collecting some benefit, but with the long-term progress accruing primarily to the cooperators.

In conclusion, the concept of stigmergy allows us to explain a broad variety of illunderstood phenomena of spontaneous coordination, in which some kind of apparently intelligent activity emerges out of simple processes. Because of its non-intuitive nature, its power of explanation is as yet poorly recognized. I hope that the present paper will inspire further researchers to pick up the thread, and examine stigmergic mechanisms in an ever-wider range of human and non-human activities. This should not only clarify deep theoretical issues, but suggest a variety of as yet unimagined practical applications.

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