Is external control important for internal control?

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Abstract

In general, systems try to have internal control; most humans want to feel like they have control over their life. Some do this by aiming for external control: they want to fix how the environment behaves. Others do this by trying to find synergies with the environment. How can we formalize these different strategies for control, and which one functions the best? I will try to answer these questions in this paper. I will do so by generalizing the concept of coordination in hierarchical systems, introduced by Mesarovic. I will apply this to model different strategies for control: a global one used in the theory of controllability, or local ones based on neural networks and perceptual control theory.

1 Introduction

A lot of people feel like they don't have control over their own life, that the path they should follow is already predetermined. They have the impression somebody or something is controlling them, and they would like to have control over their own life. But often it is presented as if you only have two choices: either dominate, or being dominated. It is assumed you should have external control in order to have internal control. Not wanting to control others, shouldn't mean to just accept everything, do nothing, don't change anything. Being passive won't bring you any closer to your liberation, nor will it lessen the control over other people.

There are different ways you can get control over your life. You can try to control everything completely, or you can try to find synergies in your environment, so that you can develop yourself to the fullest, without standing in the way of other people's development. In the first method, you keep things out of their natural state, the equilibrium state, for your own profit. You push people in a state where you want them to be, which is unnatural for them, and which they often don't want. This requires constant energy to keep this state.
Another way is that you consider that people can take different paths in life, that there are different equilibria. How their life develops depends on what they

encounter and whether they are empowered. Thus, there are different bifurcation points in one's life, and you can influence other people so that they take the path that is most aligned with you. You just interact with them so that they have the possibility to take the other path and are empowered to do so. An example is the distribution of leaflets with information about a certain oppressive practice, and ways you can resist to it. People have the choice to ignore it, but at least they now know about it, and are empowered to do something against it. This is something completely different than telling people what they should do.

An example of the different ways of control, is the difference between traditional agriculture and permaculture. In traditional agriculture, the farmer tries to control the land completely: he removes all the organisms that don't give him food directly, often just keeping one crop which he tries to optimize to get as much as possible out of it. This requires constant energy: he'll need fertilizer because the soil will get depleted from having only one crop that takes all its nutrients, and he'll need some products and machines to keep the weeds and insects away. In permaculture, the idea is that you keep the whole ecosystem. But you can have different kinds of ecosystems, depending on small differences. The idea is that you watch and learn from how nature works, and build an ecosystem where you also get out what you want. The system will maintain itself, thus it could in theory be sustained indefinitely, in a permanent agriculture, hence the name.
Another easier and more fictive example is the different ways you can deal with rain. You could just accept it, "It's raining, I'm getting wet, and I can do nothing about it". Or you could try to influence the weather. But you can also build a shelter. In this way you gain control over your life without having to control the rain.

This is related to what Peter Gelderloos[1] calls the 'Risk board mentality': the believe that contact between people who are different must result in a missionary relationship, with one converting the other. He argues that there can be a mutual influence, it's not either dominate or being dominated. You don't have to conquer the world to get control over your life.
This idea can also be related to the idea of antifragility of Taleb[2]. A system is called antifragile if it gets stronger after a shock. The principle is that you make the system more antifragile, but you don't predetermine how it should behave exactly. If you have a strict plan, a blueprint, of how everything should be, the system will be pretty fragile: as soon as something is a little bit different than planned, everything falls apart.

Similar to these different ways you can try to get control, John E. Stewart[3] defines two categories of constraints applied by management processes. Prescriptive constraints specify more or less precisely the particular outcomes that occur in the managed group. Evolvability resides in the manager, since the other entities mostly just do as they are told. With enabling constraints, the interests

of group members get aligned with the interests of the group as a whole. Then, when an agent acts in its own self-interest, it is also in the interest of the group. The advantage of this type is that it uses the local knowledge and the diversity of the group.
He also cites Salthe who states that constraints can arise in two ways: upper-level constraints arise external to the dynamic of entities, while lower-level constraints are fixed, internal features of the interacting entities that can influence how entities behave. Both influence the dynamic, but they aren't influenced in return, which is Stewart's definition of power.

I'd like to formalize this idea of different ways of control in a multi-agent model. A first difference is how local you act: you either try to control the whole system, or you can act locally to have control over your own environment. Anarchists using direct action apply this principle. Direct action means you directly act against a certain oppressive dynamic, in contrast to, for example, asking politicians to do something about it. An example is blocking an immigration detention center, so that they can't expel anyone that day.
Another difference is the way you act. You can either change your environment by adapting your links, or you can try to change your neighbors. For example, you can try to find friends who share your ideas and like what you like, or you can try to convince your friends to do what you want to do. Changing your neighbors can happen in different ways: you can let them change their goals, or you can influence them to use different methods to reach their own goal. But these three ways are related, and it might be able to put them on a continuum. Changing your links affect the possibilities your neighbors have, the methods they can use. And methods can be seen as putting a subgoal to reach a bigger goal. The question is then which goals are fundamental for an agent, and which are just means to an end. Probably this isn't that black-and-white, goals can be more or less important.

In the previous I always somehow assumed there was one agent wanting control, having power. It is important to keep in mind that being controlled doesn't necessarily have to happen by one agent. In "Perceptual control and social power"[4], McClelland argues that social power is alignment. It is when lots of people align to the same goal, that it is difficult to do something different. Power thus doesn't reside in one individual.

In the following chapters, I will try to formalize the different ways of control by using the formalization of coordination of Mesarovic[5]. He defines this for a hierarchical structure. I will first give this formalization, and then generalize it to any network. Next, I apply this formalization to special cases, where we see different ways of control. First, I show that in special cases this formalization is equivalent to the model of controllability of Liu [6]. Second, I see that if we use feedback, we can model neural networks with it. Finally, I'll use Perceptual Control Theory to build a model of agents trying to get control by changing the methods of their neighbors.

2 Coordination defined hierarchically

I now give the formalization Mesarovic[5] gives of a two-level system. I will use other symbols, because I think it will be easier to understand the model with these symbols, especially when I generalize his idea.
First, he states that a hierarchical system should have three characteristics:

1. Vertical arrangement
2. Priority of action/ right of intervention of higher level
3. Dependence of higher level upon performance lower level

He then gives three types of hierarchical systems, depending on whether it involves abstraction, decision or organization. All three can be modeled with the following model though, so I won't elaborate this further.

In his model of a two-level system (which can be easily generalized to more levels), you have a coordinator C_0, a number of infimal control systems C_i, and a process P. I call C_{ij} the coordination from C_i to C_j (where i will in general be 0, there is only coordination downwards). C_{iP} is the coordination from C_i to P. I call F_{ij} the feedback from C_i to C_j (or P if i or $j = P$). I define $C_i^+ = \bigcup_j C_{ij}$ and $F_i+ = \bigcup_j F_{ij}$, being all the coordination or feedback an agent i sends. Similarly, $C_i^- = \bigcup_j C_{ji}$ and $F_i^- = \bigcup_j F_{ji}$ is respectively the coordination and feedback i receives. Figure 1 shows a representation of this model.

Then each system is a set of functions transforming input signals into output signals. For the coordinator, this is:

$$C_0 : F_0^- \rightarrow C_0^+$$

An infimal control system makes the following transformation:

$$C_i : C_{0i} \times F_{Pi} \rightarrow C_{iP}$$

While a process works as follows:

$$P : C_P^- \times \Omega \rightarrow Y$$

where Ω is the environment, and Y the output.

We now still have to define how the feedback is generated. In an infimal system, the feedback it receives is constructed as follows:

$$f_i : C_{iP} \times \Omega \times Y \rightarrow F_{Pi}$$

The feedback the coordinator gets is:

$$f_0 : C_0^+ \times F_P^+ \times C_P^- \rightarrow F_0^-$$

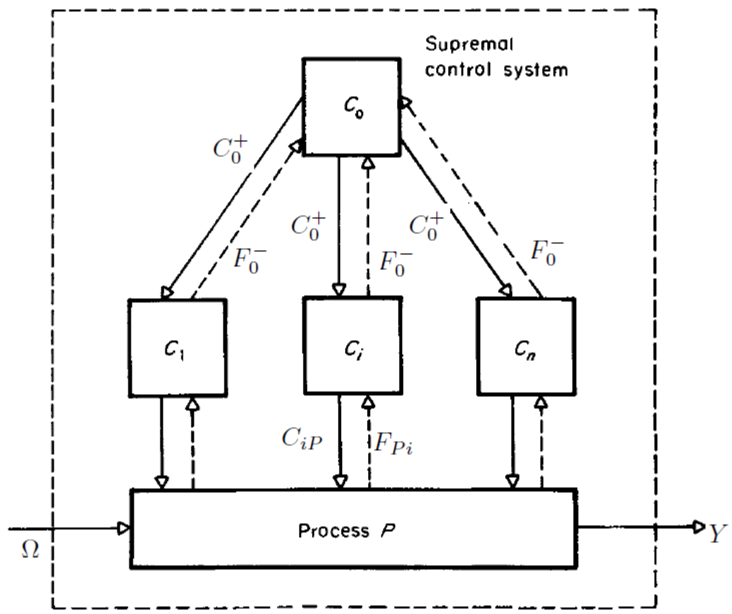

Figure 1: The model of Mesarovic

Thus what we have is that coordination always happens downstream, while feedback can only happen upstream. A system first takes all its inputs, which can be coordination inputs from the system above, feedback information from the system(s) below, or information from the environment, and transforms this into an output. This output can be coordination to the system below. Then, a system creates a feedback signal for itself by looking to the behavior of the system(s) below: which output did they generate given the inputs the system received. One of these inputs is the coordination the system has send, and the purpose of this feedback signal is to evaluate this coordination signal.

2.1 Decomposition

We can decompose these systems, to have a better understanding and intuition.

2.1.1 Decoupling

First, the system will look less complex if certain things are decoupled. It seems logical that the coordinator gets independent feedback signals from each subsystem. Thus we would get:

$$f_i0 : C_{0i} \times F_{Pi} \times C_{iP} \to F_{i0}.$$

It is possible that each subsystem controls an independent subprocess, so that the process can be decoupled.
If it isn't completely decoupled, we can add a coupling variable for such decoupling, to account for the dependencies between a decoupling.

2.1.2 Control subsystems

We can decompose each system by regarding it as a decision-making system. In general, a system S is a mapping $X \to Y$. For a decision-making system, each $x \in X$ defines a decision-problem D_x. Z is the solution set of these problems. Then there is a mapping $T : Z \to Y$. Thus, (x, y) is in the system S (or equivalent, S maps x to y), if and only if there exist a z such that z is a solution of D_x and $T(z) = y$. What this basicly does, is splitting a system into a decision unit and an implementor.
For the coordinator, this gives following mappings:

$$\begin{aligned} d_0 : F_0^- &\to X_0 \\ c_0 : F_0^- \times X_0 &\to C_0^+ \end{aligned}$$

While an infimal control system decomposes into:

$$\begin{aligned} d_i : C_{0i} \times F_{Pi} &\to X_i \\ c_i : F_{Pi} \times X_i &\to C_{iP} \end{aligned}$$

Here, the feedback signal is also used by the implementor.

2.2 Coordinability

Mesarovic also defines coordinability, in two ways: whether the infimal decision problems are coordinated relatively to the supremal decision problem or to a given overall decision problem.
Since he is only interested in the command aspect, he assumes the feedback is fixed and thus can be left out (which I think is a big assumption, a commander can also use feedback to command better). Thus there is only one supremal decision problem D_0. Further on, we define $\bar{D}(C_0^+)$ as the set $(D_1(C_{01}), ..., D_n(C_{0n}))$. Thus $\bar{x} = (x_1, ..., x_n)$ is a solution of $\bar{D}(C_0^+)$ if $\forall i : x_i$ is a solution of $D_i(C_{0i})$.
He further on defines the predicate $P(x, D)$ as:

$$P(x, D) \equiv x \text{ is a solution of } D$$

with D a decision problem.

We now have everything to give his definitions.

2.2.1 Coordinability relative to the supremal decision problem

He defines this as:

$$\exists C_0^+ \exists \bar{x} : (P(\bar{x}, \bar{D}(C_0^+)) \wedge P(C_0^+, D_0)).$$

What this means is that there is coordinability relative to the supremal decision problem if there is a coordination input from the coordinator which is a solution to its decision problem, and which gives decision problems in the infimal control subsystems that have a solution.

Note that if the d_i's and d_0 are classical functions, thus if for each C_0^+ there is one corresponding x_i, this is trivial. The decision problem D_0 then has simply one solution C_0^+. This defines the decision problems $D_i(C_0^+)$, for which each of them has the solution $x_i = d_i(C_0^+)$. Thus, then the above predicate is simply always true.
In general though, a decision problem will define a set, sometimes there will be multiple solutions, sometimes it will be the empty set if there are no solutions. Thus the above is true if D_0 has some solution(s), and one of these solutions defines $D_i(C_0^+)$'s that all have a solution.

2.2.2 Coordinability relative to a given overall decision problem

We can assume that the overall decision problem depends on the process, thus that we want to control the coordination signals to the process, C_P^-. Thus the solutions of the overall decision problem D will be in the set C_P^-. Since the feedback is fixed, the implementor of the infimal systems becomes $\pi : X \to C_P^-$, with $X = \bigcup_i X_i$, the solutions of all infimal decision problems. Then we can define coordinability relative to an overall decision problem as:

$$\exists C_0^+ \exists \bar{x} : (P(\bar{x}, \bar{D}(C_0^+)) \wedge P(\pi(\bar{x}), D)).$$

Thus the system is coordinated if there is a coordination input from the coordinator such that the infimal decision problems have a solution and this solution transforms in a solution for the overall decision problem.

3 Generalization

I'd like to generalize this model to any kind of network, where there isn't necessary a 'top' and a 'bottom'. I also won't differentiate anymore between a process and a control system, everything is simply an agent. An agent gets certain input, this can come from other agents in the form of coordination or feedback, or could come from 'outside', from the environment. It transforms this into an output, which can be a coordination input for other agents, or some general output going outside.
For the feedback, we assume the feedback an agent receives is decoupled, thus it gets several feedback signals from the agents he send a coordination input

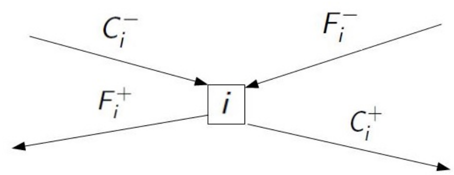

Figure 2: Inputs and outputs of an agent in the general model

to. The function to create a feedback input will thus be constructed from the sending perspective. An agent uses his inputs and outputs to create a feedback signal.

We'll work with a directed network. The inputs and outputs of an agent i are shown in figure 2. A link between agent i and agent j, means i sends a coordination input C_{ij} to j. Consequently, j sends a feedback signal F_{ji} to i. Further on, it's possible an agent receives some environment input Ω_i and send some output Y_i outside. I'll consider these also as coordination inputs and outputs, thus I define $C_i^- = \Omega_i \cup \bigcup_j C_{ji}$ and $C_i^+ = Y_i \cup \bigcup_j C_{ij}$. Then we get following functions:

$$C_i : C_i^- \times F_i^- \to C_i^+$$
$$F_i : C_i^- \times F_i^- \times C_i^+ \to F_i^+$$

An agent uses its coordination and feedback inputs to create a coordination output, and takes all of this to generate a feedback signal.

I now would like to define coordinability in this framework. For the sake of simplicity, first I will assume there is no feedback.

3.1 Coordinability

We can again decompose C_i into two subsystems. Since there is no feedback, this gives:

$$d_i : C_i^- \to X_i$$
$$c_i : X_i \to C_i^+$$

Now, we can't define coordinability anymore relative to a supremal decision problem, since that doesn't exist anymore. But we can say a system is coordinated if its agents are coordinated. A definition of **weak internal coordinability** would be:

$$\exists \bar{c}, \bar{x} : \forall\ i P(x_i, D_i(C_i^-))$$

where $\bar{c} = \bigcup_{i,j} C_{ij} \cup \bigcup_i \Omega_i \cup \bigcup_i Y_i$, and $\bar{x} = (x_1, ..., x_n)$.

This means we can find coordination inputs so that each decision problem has a solution. If the d_i's are functions (thus every decision problem has one solution), this is again always true. We would like however that the coordination inputs are constructed according to our model, thus from a previous $x_i(t-1)$, by the formula $C_i^+(t) = c_i(x_i(t-1))$. But it's not necessary that the $x_i(t)$ that is a solution of $D_i(C_i^-)$ is the same as the $x_i(t-1)$ used to build this C_i^-. In a predicate, this looks like:

$$\exists \bar{x}(0), \exists t : \forall\ i P(x_i(t), D_i(C_i^-(t))) \wedge c_i^+(t) = c_i(x_i(t-1))$$

This adds a time parameter to our model though, and in practice it is difficult to check whether this holds. What we can look at though, is whether the following is true:

$$\exists \bar{c}, \bar{x} : \forall\ i P(x_i, D_i(C_i^-)) \wedge c_i^+ = c_i(x_i)$$

This assumes some stability: the same x_i that is used to build the coordination outputs, should be a solution to the decision problem defined by the coordination inputs. Notice that C_i^- is uniquely defined by x_i, thus we don't really have to search for the right C_i^-, so the following is equivalent as the above:

$$\exists \bar{x} : \forall\ i P(x_i, D_i(C_i^-)),\ \text{with}\ C_i^+ = c_i(x_i)$$

I call this **stable internal coordinability**. If each decision problem has one solution, d_i is a function, this is equal to:

$$\exists \bar{c}, \bar{x} : \forall\ i\ x_i = d_i(C_i^-) \wedge C_i^+ = c_i(x_i)$$

or, using the composed version again:

$$\exists \bar{c} : \forall i\ c_i^+ = C_i(c_i^-)$$

Now, we can also define coordinability relative to a given problem D. We can assume the solutions to this problem are in $Y = \bigcup_i Y_i$. There is a function $\pi : X \to Y$ (part of the c_i's). I again split up in a weak and a stable version.

There is **weak coordinability relative to a problem D** if:

$$\exists \bar{c}, \bar{x} : \forall\ i P(x_i, D_i(C_i^-)) \wedge P(\pi(\bar{x}), D)$$

While **stable coordinability relative to a problem D** is defined as:

$$\exists \bar{x} : \forall\ i P(x_i, D_i(C_i^-)) \wedge P(\pi(\bar{x}), D),\ \text{with}\ C_i^+ = c_i(x_i)$$

I would now like to extend these definitions so that it also allows feedback.

3.1.1 with feedback

The weakest version of coordinability here would be:

$$\exists \bar{c}, \bar{f}, \bar{x} : \forall\, iP(x_i, D_i(c_i^-, f_i^-))$$

Here, we can simply choose the coordination and feedback arbitrary, which doesnt really add any value to using feedback. A stronger version is:

$$\exists \bar{c}, \bar{f}, \bar{x} : \forall\, iP(x_i, D_i(c_i^-, f_i^-)) \wedge c_i^+ = c_i(f_i^-, x_i)$$

Thus here the coordination input should be constructed from some feedback and the same solution of the decision problem, implying already some stability. But the feedback can be chosen arbitrary, thus this is pretty similar as the case without feedback. To take feedback into account, look at the following predicate:

$$\exists \bar{x}, \bar{c}, \bar{f} : \forall i\; P(x_i, D_i(c_i^-, f_i^-)) \wedge f_i^+ = F_i(c_i^-, f_i^-, c_i^+)$$

This gives us a feedback version of **weak internal coordinability**. Thus here, the x_i should be a solution of the decision problem defined by the coordination and feedback inputs, while the feedback should be constructed from the feedback and coordination inputs it received, and the coordination output it sends. There is thus already some stability here, on the level of the feedback. The feedback it sends out shouldn't change the feedback signal of its neigbours. The coordination inputs can still be chosen arbitrary, though we would like them to be constructed from a previous $x_i(t-1)$.

Stable internal coordinability is in case of feedback defined as:

$$\exists \bar{x}, \bar{f} : \forall i\; P(x_i, D_i(c_i^-, f_i^-)) \wedge f_i^+ = F_i(c_i^-, f_i^-, c_i^+),\ \text{with } c_i^+ = c_i(f_i^-, x_i).$$

If the d_i's are functions, this is equivalent to:

$$\exists \bar{c}, \bar{f} : \forall i\; c_i^+ = C_i(c_i^-, f_i^-);\; f_i^+ = F_i(c_i^-, f_i^-, c_i^+) \tag{1}$$

We can similarly speak of weak or stable coordinability relative to a problem D, by adding the condition $P(\pi(\bar{x}), D)$.

4 Application to the controllability of complex networks

I would now like to apply this framework to the theory of the controllability of complex networks[6]. The idea here is that you try to control a network by sending certain inputs to certain nodes. You would like to find a minimal set of nodes which you have to control so that you control the whole network. In their model, each node j has a value X_j which got influenced by the values of their neighbours and the control input. This happens by the equation:

$$X_j = X_j + \sum_i a_{ji} X_i + \sum_k b_{jk} u_k$$

where a_{ji} is the link weight between X_i and X_j, and b_{jk} is the link weight from the controller u_k to X_j. They argue that the exact values of a_{ji} and b_{jk} don't matter for the controllability. We can write this into our framework by taking X_j, the output of j, also as input of j. We take $a_{ii} = 1 \forall i$. An agent sends the same output to all agents. We got:

$$C_i : C_i^- \times \Omega_i \to C_i^+$$
$$X_i = \sum_j a_{ij} X_j + \sum_k b_{ik} u_k = C_i(X, U)$$

where $X_j \in C_i^-$ and $u_k \in \Omega_i$.

They define controllability as being able to put the network in any desired state. It isn't necessary however that this is a steady state. You can try to steer to this state by choosing certain inputs. The problem to define it is similar to the problem we faced in defining coordination. It isn't necessary that it is a stable state, but we would like the coordination inputs to be not just random, but constructed from a previous iteration. We can define **weak controllability** as a form of weak coordinability, namely when there is weak coordinability relative to all decision problems, thus

$$\forall D \exists \bar{x} : \forall\, i P(x_i, D_i(C_i^-)) \wedge P(\bar{x}, D)$$

Notice that the implementer is here the identity function; the solution of the decision problem x_i gets send out. d_i is a function, thus the first part is actually always true, though we would like the C_i^- to come from a previous step. The second part states that we should find an $\bar{x}$ that is a solution to the decision problem, for all the decision problems. A decision problem is a subset of possible $\bar{x}$'s, thus this is equivalent of stating it's true for all $\bar{x}$'s. The theory of controllability found out that you have controllability if each node has its own direct superior, this can be another node or an input node. This leads them to the theorem that a minimal set of nodes you have to control is equal to the unmatched nodes in a maximum matching.

The requirement that you should reach a stable state could however be useful. Reaching a desired state for only a millisecond, is often not what you want. I thus define **stable controllability** as:

$$\forall \bar{c} \exists \bar{u} : \forall i \; c_i^+ = C_i(c_i^-, \bar{u}).$$

We can again define this by seeing it as an overall decision problem. The solutions of the decision problem are in $\bar{C}$. It is a solution of the problem if it is equal to our predefined desired state. Thus controllability means the following is true for all decision problems:

$$\exists \bar{c}, \bar{u} : \forall i \; c_i^+ = C_i(c_i^-, \bar{u}) \wedge P(\bar{c}, D).$$

Internal stable coordinability is here defined as:

$$\exists \bar{c}, \bar{u} : \forall i \; c_i^+ = C_i(c_i^-, \bar{u})$$

which is thus less strong - there only need to be one stable solution.

I'd now want to check whether there is stable controllability in this model. Thus we consider X_i fixed for all i. We find that we want to find u_k's such that

$$\sum_i a_{ji} X_i + \sum_k b_{jk} u_k = 0 \forall j$$

where we don't consider a_{jj}, thus we put this at 0 again. Define

$$S_j = -\sum_i a_{ji} X_i$$

(this is completely defined, since X_i and a_{ji} are given.) Then we find we should have

$$\sum_k b_{jk} u_k = S_j \forall j$$

If for a certain j, $b_{jk} = 0 \forall k$, then we should have $S_j = 0$. Otherwise, we should define one u_l as depending on the others by the formula:

$$u_l = \frac{S_j - \sum_{k \neq l} b_{jk} u_k}{b_{jl}}$$

Thus, each node for which $S_j \neq 0$ should have its own control input, which means almost all nodes should be controlled.
If we take $X_i = 0 \forall i$, we find a solution for stable internal coordinability.

5 Self-organized control

I would now like to extend the above model to allow feedback. The idea is that we see the feedback as link weight. The link weight is changed so that the input an agent receives fulfills his desire more. This is thus a model where an agent changes his environment in order to get control. I model this by giving each agent a reference value R_i. An agent wants to move its value X_i to the reference value. The updating of a value of an agent happens as above, except that I don't allow any external input anymore. We consider the coordination an agent send as its value multiplied with the link weight (this operation thus happens with the sending agent instead of with the receiving agent). I consider two loops: an agent sends its updated value and the constructed feedbacks also to himself, so that it's first output, and then gets input. I thus get:

$$
\begin{aligned}
C_i : \underbrace{X_i}_{\in C_i^+} &= \underbrace{X_i}_{\in C_i^-} + \sum_j \underbrace{C_{ji}}_{\in C_i^-} \\
\underbrace{C_{ij}}_{\in C_i^+} &= \underbrace{F_{ji}}_{\in F_i^-} \underbrace{X_i}_{\in C_i^-} \\
F_i : \underbrace{F_{ij}}_{\in F_i^+} &= F_{ij} + \alpha(R_i - X_i)X_j \\
&= \underbrace{F_{ij}}_{\in F_i^-} + \alpha(R_i - \underbrace{X_i}_{\in C_i^+}) \underbrace{C_{ji}}_{\in C_i^-} / \underbrace{F_{ij}}_{\in F_i^-}
\end{aligned}
$$

The last formula for the updating of the link weight comes from the theory of perceptron learning [7]. If the total input is too big, the link weight (feedback) is weakened for positive inputs, and strengthened for negative ones, so that the total input becomes less. The opposite happens if the total input is too little. Figure 3 shows how these functions work.

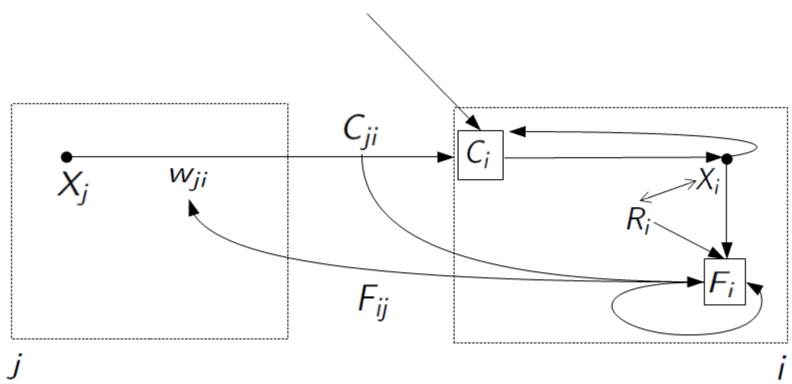

Figure 3: Self-organized control

I'd now like to know whether there is stable internal coordinability in this model. This is the case if there is a solution for the above equations (from (1)). Thus, if:

$$
\begin{aligned}
\sum_j C_{ji} &= 0 \\
C_{ij} &= F_{ji} X_i \\
\alpha(R_i - X_i)X_j &= 0
\end{aligned}
$$

for all i. A solution for this is $X_j = 0 \forall j$, then the feedback can be chosen at random. Another possibility is to take $X_i = R_i \forall i$. Then we should have

$$\sum_j F_{ij} R_j = 0 \forall i$$

Consider a particular i. If $R_j = 0 \forall j$, it's ok. Assume there is a $k : R_k \neq 0$. For $j \neq k$, we can then take F_{ij} at random, and then take

$$F_{ik} = \frac{-\sum_{j \neq k} F_{ij} R_j}{R_k}$$

Since we can do this for all i, we find another possibility for stable internal coordinability. It seems most logical that the agents wants to have their values equal to their reference values. We can put this as an overall decision problem: $R_i = X_i \forall i$. Then we get that the above solution is the only possible case of stable coordinability relative to this overall decision problem.

Thus we find that there is stable coordinability in this model, if we assume the feesback (link weight) isn't bound to only positive numbers or only between 0 and 1. Notice though that not because there is a solution, that this solution will necessary be reached by this process. It might never get into this attractor. For example, if we take $X_i = 0 \forall i$, none if the X_i's will change, thus it won't be able to reach it's reference value if this value is different than zero. Also, if the learning parameter α is too high, agents might constantly overcompensate, thus never reaching the reference value.
Thus, also in a weak version, coordinability won't always be reached. In which circumstances it is and isn't, remains an open question, though it is suspected that often it is reached, since the model is build so as to go to the solutions.

6 Control by changing the method of your neighbors

I now want to construct a model where agents try to influence the methods of their neighbors. I will base this on Perceptual Control Theory[8].

In perceptual control theory, an agent tries to control its perception, by trying to equalize it with a certain reference value. But there are also other agents who try to control the same perception, with other reference values in mind, and the perception might also get disturbed by the environment. I is however that these disturbances are random, thus this isn't much of a problem. The model thus looks like this:

$$C_i = C_i + \alpha(R_i - X_i)$$
$$X_i = \sum_j C_j + \Omega_i$$

In this model, the perception will converge to an average of the reference values, where α represents the power an agent has. This can thus be a model of social power. If all the other agents have the same reference value, the one (or minority of) agent(s) with another reference value won't be able to match its reference value. In general, this one control loop is part of a perceptual hierarchy, with different reference values which are controlled by a higher control loop. If there is conflict because a reference value is constantly not met, it will perturb this reference value until it is met. Thus, an agent will conform to the pressure of the group of all the same reference values.

I now want to construct a model based on this where the idea is that you try to influence your neighbors to send the right coordination input, by sending them certain feedback. We assume the coordination an agent j sends to i is constructed as follows:

$$C_{ji} = C_{ji} + F_{ij}$$

Each agent i has a value X_i he wants to put as close as possible to its reference value R_i. X_i is constructed as follows:

$$X_i = \sum_j C_{ji}$$

The way an agent tries to control its neighbors to send coordination which satisfies its needs (reference value) more, is by sending this feedback:

$$F_{ij} = \alpha(R_i - X_i)$$

The shortcoming of this model that the coordination input is assumed to be known and of a specific form, which satisfies our urge to be able to control it. That's why I want to generalize the model where we assume the coordination function is unknown, and we don't even know how exactly this gets aggregated in to X_i. Thus we just assume

$$X_i = f(C_i^-)$$

with f some unknown function. Then we can still try to get control by looking how our X_i got affected by the F_i^+ we have send out. If there is a positive correlation, thus a bigger F_i^+ results in a bigger X_i, then we can use the same update mechanism as above. If there is a negative correlation however, we should do the opposite, subtracting instead of adding. We thus get the following formula's:

$$\begin{array}{lll} F_i^+ = F_i^+ + \alpha(R_i - X_i) & \text{if} & F_i^+ \nearrow \;\Rightarrow\; X_i \nearrow \\ F_i^+ = F_i^+ - \alpha(R_i - X_i) & \text{if} & F_i^+ \nearrow \;\Rightarrow\; X_i \searrow \end{array}$$

This still has some unrealistic assumptions though, because the aims and the methods got separated. The X_i and the output C_{ij} you send out are completely separated. We see what kind of consequences this has when we check whether there is stable internal coordinability. It's difficult to check this for the general model, so I do this for the more specific model. There is stable internal coordinability if

$$\begin{aligned} 0 = F_{ij} &= \alpha(R_i - X_i) \\ &\Rightarrow R_i = X_i \end{aligned}$$

And

$$R_i = X_i = \sum_j C_{ji}$$

You can easily choose C_{ji}'s such that this is fulfilled.

7 Conclusion

I have examined different models of control here. We can put this in my framework of the scope and the way you act.

The first group of models try to control a whole network, they work globally. The model of controllability is an example of this. Another example is the model of Emergent Control[9], where they try to achieve a global goal by adapting the local rules. We saw that at least in the model of controllability, this is difficult to achieve because you have to control almost all the nodes, pushing it away from its natural state.

Other models work locally, they assume the agents want to get control. In my first model of self-organized control, they did this by changing the links they had with other neighbors. This worked, under the assumption that the feedback (link weights) wasn't bound too much. In the second model of this kind, agents tried to adapt the methods of their neighbors. This also worked, but there was the implicit assumption that goals and methods are separated, which isn't very realistic.

A general shortcoming of these models is that we assume the goal of an agent is simply to reach a certain reference value, while in reality goals are usually far more implicit and multidimensional. It might even be better to speak about certain value systems instead of certain goals, where there isn't one optimal solution. But this is more difficult to formalize, and the general principles gotten out of this paper seem to be also true in this case. I.e. that it's easier to get control over your life by acting locally, as least as possible disturbing the core values of your environment. It might even be more easy to do so in reality, because there are far more possibilities to satisfy your values.

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